OBJECTIVES

After completing this exercise, the student will be able to:

1. draw a simple ellipse using two thumbtacks and a loop of string.
2. calculate the eccentricity of an ellipse.
3. use a series of lunar photographs to make a scale drawing of the moon’s orbit.
4. be able to confirm the elliptical nature of the moon’s orbit by determining the major axis, minor axis, and eccentricity of the orbit.

STUDENT MATERIALS

- pencil
- mm ruler
- compass
- protractor
- calculator


LAB MATERIALS

- two thumbtacks per student
- one loop of string per student
- a piece of cardboard (∼8 1/2×11") per student

STUDENT REQUIREMENTS

This lab is to be done individually without lab partners. After completing the lab, turn in Table I, Fig. 1, and Fig. 2 in page order sequence.

PROCEDURE

A. Kepler’s Laws and the Elliptical Orbit

You will draw an elliptical orbit on the page provided (Fig. 1). To do this you will need a piece of cardboard, two thumbtacks, and a loop of string.

1. Remove Fig. 1 and place it on the piece of cardboard.
2. Insert a thumbtack through the two focus points labeled F1 and F2.
3. Make a loop of string which is large enough to go around both tacks at the same time. The loop should stretch about 8 to 10 cm, but its actual size is not important.
4. Place the loop of string around the tacks and stretch it taut with a sharp pencil point as demonstrated by your instructor. While holding the pencil vertically and keeping the string taut draw an ellipse by moving the pencil around the two tacks. The orbit you have drawn is similar to that of some comets and asteroids.
5. Draw a line longwise through the ellipse which connects F1 and F2. This line is called the major axis. Label the major axis on your drawing.
6. The minor axis is perpendicular to the major axis and divides it in half. The midpoint of your major axis is halfway between F1 and F2. Locate this point and label it c. Now draw and label the minor axis on your ellipse drawing.
7. Kepler’s 1st law states that the Sun is at one focus of an elliptical orbit. Pick one of the foci and label it Sun. The other focus has no physical significance.
8. Perihelion is the closest approach to the Sun. Aphelion is the farthest point from the Sun. Perihelion and aphelion are on the orbit at each end of the major axis. Find and label these two points on your orbit.
9. The distance from the center c to either perihelion or aphelion is named the semi-major axis. Label this distance as a on your orbit. The distance from the center to one of the foci is named x in this exercise. Label this distance on your orbit.
10. On your orbit, measure, in AUs (1 cm = 1 AU) the perihelion distance, the aphelion distance, and the major axis, the semi-major axis, and the distance, $x$. Record your results to a tenth of an AU in the spaces provided below your drawing.

11. How much an ellipse deviates from a circle is called its eccentricity, $e$. Calculate your orbit's eccentricity using the relation $e = x/a$, and record your answer in the space provided. Note that eccentricity is a ratio and has no units.

12. Kepler's 3rd law states that the orbital period, $P$, is related to the orbit's semi-major axis, $a$, by the relation $P^2 = a^3$, where $P$ will be in years if $a$ is in AUs. Use the 3rd law to calculate the period of your orbit. Record your answer to the nearest tenth of a year in the space provided.

### B. The Moon's Orbit

As applied to the moon’s orbit, Kepler’s 1st law would state that the moon’s orbit is an ellipse with the Earth at one focus. If the moon’s orbit is an ellipse its distance from the Earth should change during one complete orbit. This means the moon should appear larger at perigee (closest approach to Earth) than it does at apogee (farthest from Earth). The *Sky and Telescope* reprint provided by your instructor shows a series of lunar photographs. These were obtained as the moon passed several different positions in its orbit. It is possible to use these photographs to determine a few orbital properties of the moon.

1. Measure the diameter, $d$, of each lunar image shown. Be careful not to simply measure the illuminated portion. Make all measurements more or less vertically. Estimate each value to a tenth of a millimeter using a magnifier. Record your measurements in the column provided in Table I. (Note: these values should range between 40 mm and 60 mm.)

2. From each diameter measurement the relative distance to the moon can be determined. This distance will be scaled down to fit on a piece of graph paper. Calculate the scale distance, $D$, (in mm) to the moon using the relation $D = 4000/d$, where $d$ is the diameter recorded in column 4 of Table I and 4000 is a scale factor. Record your values of $D$, to its nearest tenth of a millimeter, in the last column of Table I.

3. Calculate the average scale distance, $D_{ave}$, to the moon and record your answer in the space provided at the bottom of Table I.

4. Plot the moon's orbital position on the polar coordinate graph paper given as Fig. 2. You will have to plot longitude vs. scale distance, $D$. On the graph paper longitude starts at the bottom, labeled $0^\circ$, and increases counterclockwise. This will correspond to the true direction of the moon's orbital motion as viewed from above the Earth's north pole.

To plot the first data point, place the edge of your millimeter ruler along the line labeled $270^\circ$ (longitude). Be sure that 0 mm is at the origin of the graph paper, which corresponds to the Earth's position on this diagram. Now measure out along the $270^\circ$ line and place a pencil dot at your value of $D$ for a longitude of $270^\circ$. Continue in this manner for the remaining data in Table I.

5. If you examine the plotted data on Fig. 2 it can be seen that they are not centered around the Earth. In order to draw the moon's orbit you must first establish the orbit's center. Open your compass equal to the $D_{ave}$ calculated in step 3. Remember to use the edge of a ruler to properly set the compass. Locate the center by placing the compass point on three or four of the plotted data points and draw a small arc near the center of the graph. Ideally these should all cross at one place, the center. In reality, they will show you only approximately where the center should be.
6. Place the compass point within the small area enclosed by your arcs drawn in part 5. Check to see how well the compass pencil passes through the data points. This will not be perfect. Adjust the compass point’s location until a best-fit circle can be drawn. Once you have located the center, draw in the moon’s orbit with your compass. You are thus representing the elliptical orbit of the moon with an offset circle instead of an ellipse.

7. The compass hole is the center of the moon’s orbit. One focus is located at Earth, which is at the origin of the graph paper. Use these two points to draw and label the major and minor axes of the moon’s orbit. Hint: How are the center and foci related to the major and minor axes in Fig. 1?

8. Label perigee (closest) and apogee (farthest) on the moon’s orbit. These may, or may not, be at data point locations.

9. Measure the values of x and a in millimeters. Calculate the eccentricity of the orbit using \( e = \frac{x}{a} \). Record your results in the spaces provided on the graph paper.

   Turn in Fig. 1, Table I, and Fig. 2.
Fig. 1: Drawing of an elliptical orbit.

Major Axis = e
Aphelion = a
Perihelion = p

F₂

F₁

SECTION

NAME
TABLE I

MOON'S ORBIT

<table>
<thead>
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<th>IMAGE</th>
<th>DATE (Nov.)</th>
<th>LONGITUDE ('')</th>
<th>d (mm)</th>
<th>D (mm)</th>
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\[D_{\text{avg}} = \underline{\text{_____}}\]