THE MASS OF JUPITER

4

OBJECTIVES

The objectives of this activity are intended to meet the following course goals:

G1. Students will learn basic astronomical laboratory skills and understandings.
G8. Students will learn the big concepts of astronomy.

After completing this lab the student will be able to:

1. observe the motions of Jupiter's moons.
2. construct a graph position vs. time for one of the Galilean moons.
3. use Kepler's 3rd Law to determine the mass of Jupiter based on observations of the Galilean moons.

STUDENT MATERIALS

The student should bring the following items to lab class:

pencil
calculator

LAB MATERIALS

- CLEA software, The Revolutions of the Moons of Jupiter, or some other appropriate software.
- Computers

STUDENT REQUIREMENTS

Data may be collected by individuals or by pairs of students. All data reductions and analysis, graphs, and answers to questions should be completed by individuals, **NOT** by pairs or groups of students. Turn in **Table I**, **Fig. 2a**, **2b**, **2c**, **2d**, and your answers for Questions and Calculations section of this lab.

INTRODUCTION

During 1609 and 1610 the Italian astronomer Galileo Galilei was the first person to use a simple telescope to observe the moons of Jupiter. He noted that the Jovian moons appeared to behave like a miniature Solar System with Jupiter replacing the Sun and the moons playing the role of planets. In part this helped to give support to the Copernican Heliocentric Model of the Solar System, because these moons did not orbit the Earth. The center of their system was Jupiter. So not everything orbited the Earth or had the Earth at its center. About this same time a German mathematician and astronomer, Johannes Kepler, was developing his three laws of planetary motion. The four Galilean satellites of Jupiter appeared to obey these laws.

About 50 years later, in 1666, Issac Newton developed his Law of Gravity and revised Kepler's three Laws of Planetary Motion. Kepler's 3rd Law was rewritten to include the masses of the two orbiting bodies. Mathematically, this can be written as

$$\mathbf{P^2 (M_1 + M_2) = a^3} \qquad \textbf{(Eq. 1)}$$

where: **P** = orbital period in years,

a = semi-major axis or the radius of the orbit in AUs,

$\mathbf{M_1}$ and $\mathbf{M_2}$ = masses of the two orbiting bodies in Solar masses.

Rearranging terms, **Eq. 1** can be solved for the masses $\mathbf{M_1}$ and $\mathbf{M_2}$,

$$\mathbf{(M_1 + M_2) = a^3/P^2} \qquad \textbf{(Eq. 2)}$$

Notice that **Eq. 2** gives the total mass of the two orbiting objects, not the individual masses of each body. In the case of a large planet and a tiny moon, such as Jupiter and one of its moons, it can be assumed that the satellite has almost no mass relative to the very massive planet. Thus, $\mathbf{M_2 = 0}$, **Eq. 2** becomes

$$\mathbf{M_1 = a^3/P^2} \qquad \textbf{(Eq. 3)}$$

and M_1 is effectively the mass of the planet.

During this lab exercise you will use software to simulate observations of Jupiter and its four Galilean satellites as viewed with a small Earth-based telescope. Data will be collected on the orbit of one Jovian moon and the mass of Jupiter will be calculated using **Eq. 3**. The orbiting moons are really fun to watch, so enjoy yourself while doing this simulation.

OBSERVING PROCEDURE

Your lab instructor will assign each group of students a Galilean Moon to observe using the software provided. Io is rather close to Jupiter and only takes a couple of days to orbit the planet. At the other extreme is Callisto, which is relatively far from Jupiter and takes about three weeks for one revolution. Thus the time intervals necessary to observe each moon through a complete orbit is different for each satellite. Io must be observed every few hours for two or three days and Callisto should be observed once per day for several weeks. The frequency with which you should make observations for your assigned moon is given below:

Io: Observe every 2 hours for 3 days.

Europa: Observe every 4 hours for 6 days.

Ganymede: Observe every 6 hours for 9 days.

Callisto: Observe twice per day for 18 days.

DATA REDUCTIONS AND ANALYSIS

1. Make a graph of J.D. vs. Time for your assigned moon on **Fig. 2a**, **2b**, **2c**, or **2d**, whichever corresponds to your satellite. The time scales are in hours. The beginning of a new day is represented by a large tick mark. Label these large tick marks with the dates of your observations as shown on the example graph of **Fig. 1**.

2. After plotting all the data, draw a best-fit curve through the data. This curve should be smooth and free of any kinks or sudden jogs. See **Fig. 1** for an example of how your curve should look.

3. Use the curve you have just drawn in **Fig. 2** to determine the orbital period, P, for your satellite. In order to use Kepler's 3rd Law, you will need the period in units of years. However, it will probably be easier to determine the period in hours or days and convert these into years. Record your value for P in the spaces provided on **Fig. 2**. See **Fig. 1** as an example of how to determine P.

4. The amplitude of the curve in **Fig. 2** represents the orbit's diameter, or major axis. To use Kep-ler's 3rd law, you need the value of the orbit's semi-major axis, a. Because the orbit is elliptical and because we view the orbit from varying locations, you need to measure the full amplitude of the curve (top to bottom) as shown in **Fig. 1**. this is the orbit's diameter or 2a, so divide by 2 to get the orbit's semi-major axis, a. Record this value in the space provided in **Fig. 2**. In order to use Kepler's 3rd Law, you must convert this value into AUs by multiplying it by 9.545×10^{-4} AU/JD. Record your answer in the space provided in **Fig. 2**.

Table I

Jupiter's Diameter = 7 mm

Asigned Moon is Io

Date	Time	Distance to moon in mm's	Distance to moon in JD's
8/26/95	01:40	+8	+1.1
	03:25	+4	+0.6
	05:30	-2	-0.3
	07:30	-7	-1.0
	09:30	-11	-1.6
	11:35	-14	-2.0
	13:40	-18	-2.6
	15:30	-18	-2.6
	17:25	-17	-2.4
	19:30	-14	-2.0
	21:35	-11	-1.6
	23:30	-8	-1.1
8/27/95	01:25	-2	-0.3
	03:30	+4	+0.6
	05:25	+9	+1.3
	07:45	+13	+1.8
	09:40	+16	+2.3
	11:30	+18	+2.6
	13:25	+18	+2.6
	15:30	+16	+2.3
	17:35	+13	+1.8
	19:30	+9	+1.3
	21:30	+4	+0.6
	23:35	-1	-0.1
8/28/95	01:35	-5	-0.7

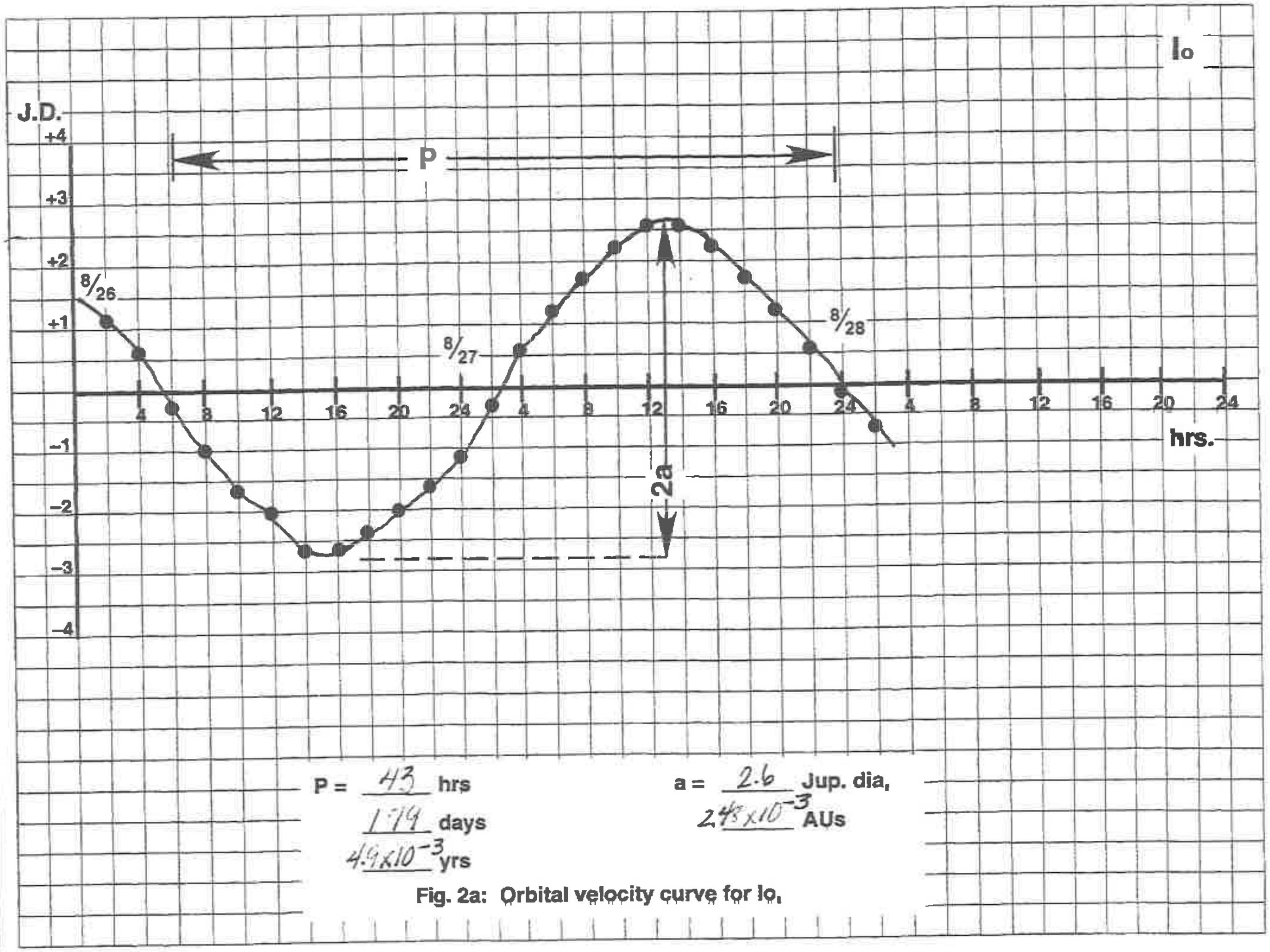


Fig. 2a: Orbital velocity curve for Io.

Fig. 1: An example of the results obtained for Io.

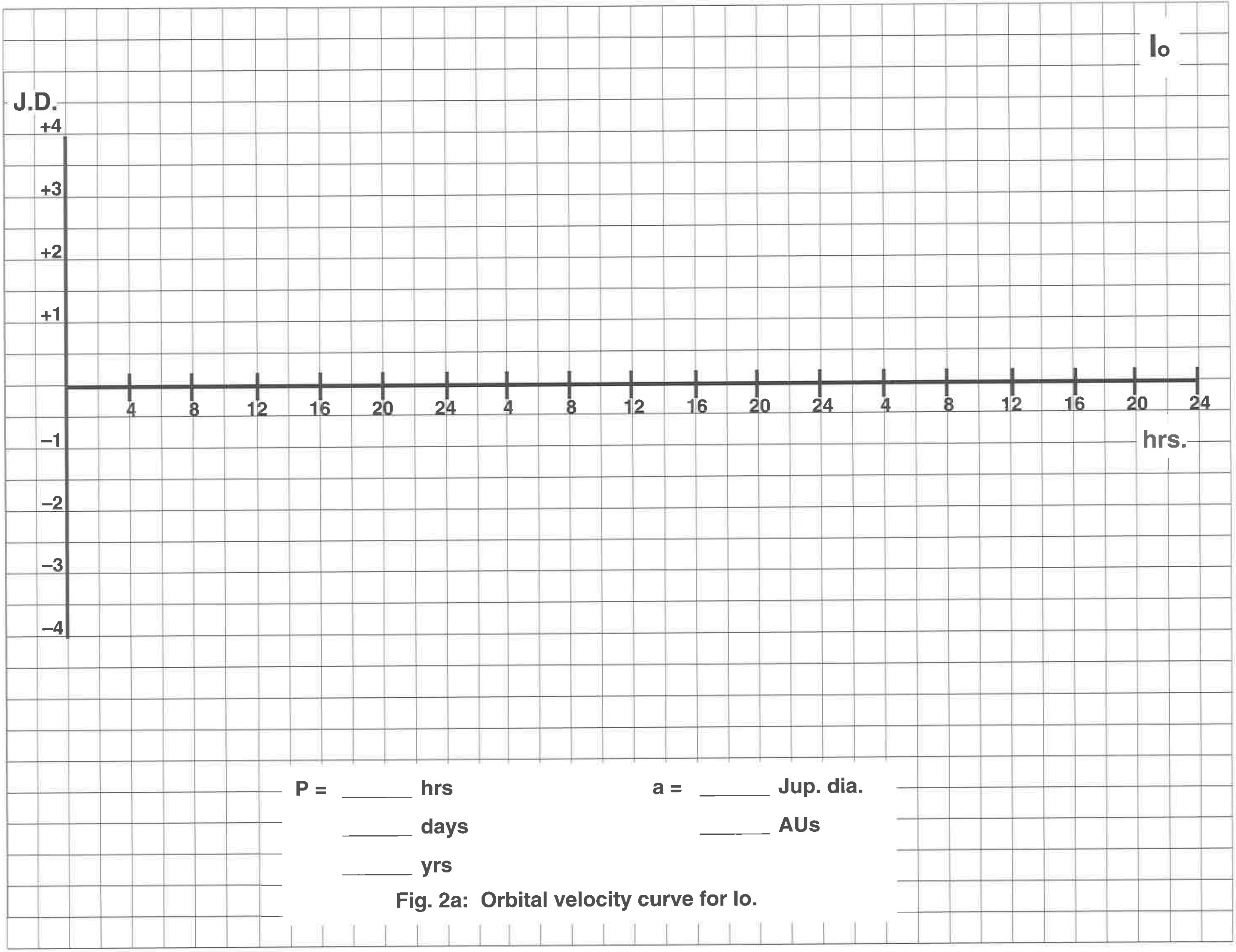
NAME: ______________________________

SECTION: ______________________________

Table I

Assigned Moon ______________

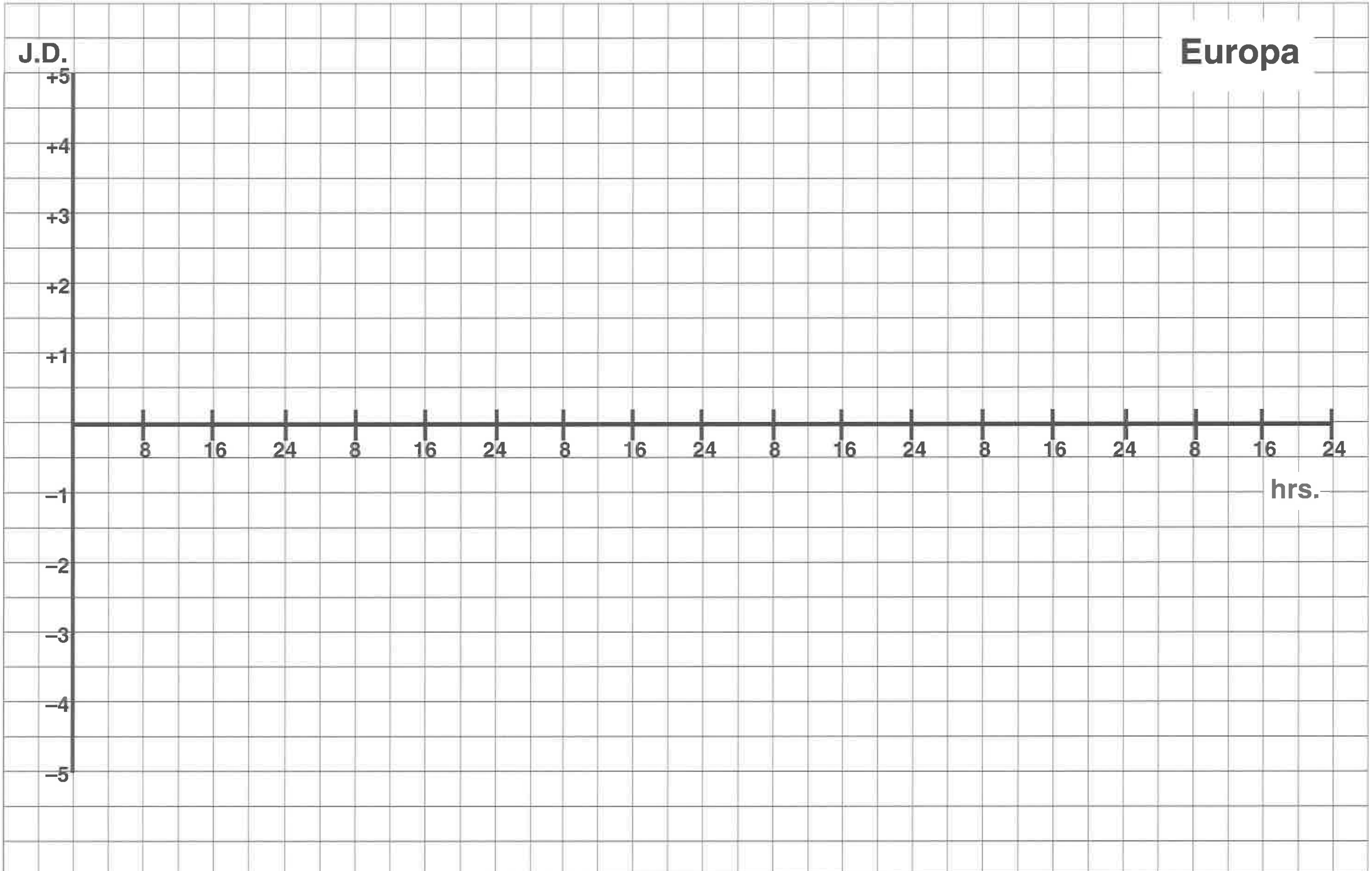
Date Month/day	**Time** H:M	**Distance from Jupiter in J.D.s** **E (-) and W (+)**



P = _______ hrs
_______ days
_______ yrs

a = _______ Jup. dia.
_______ AUs

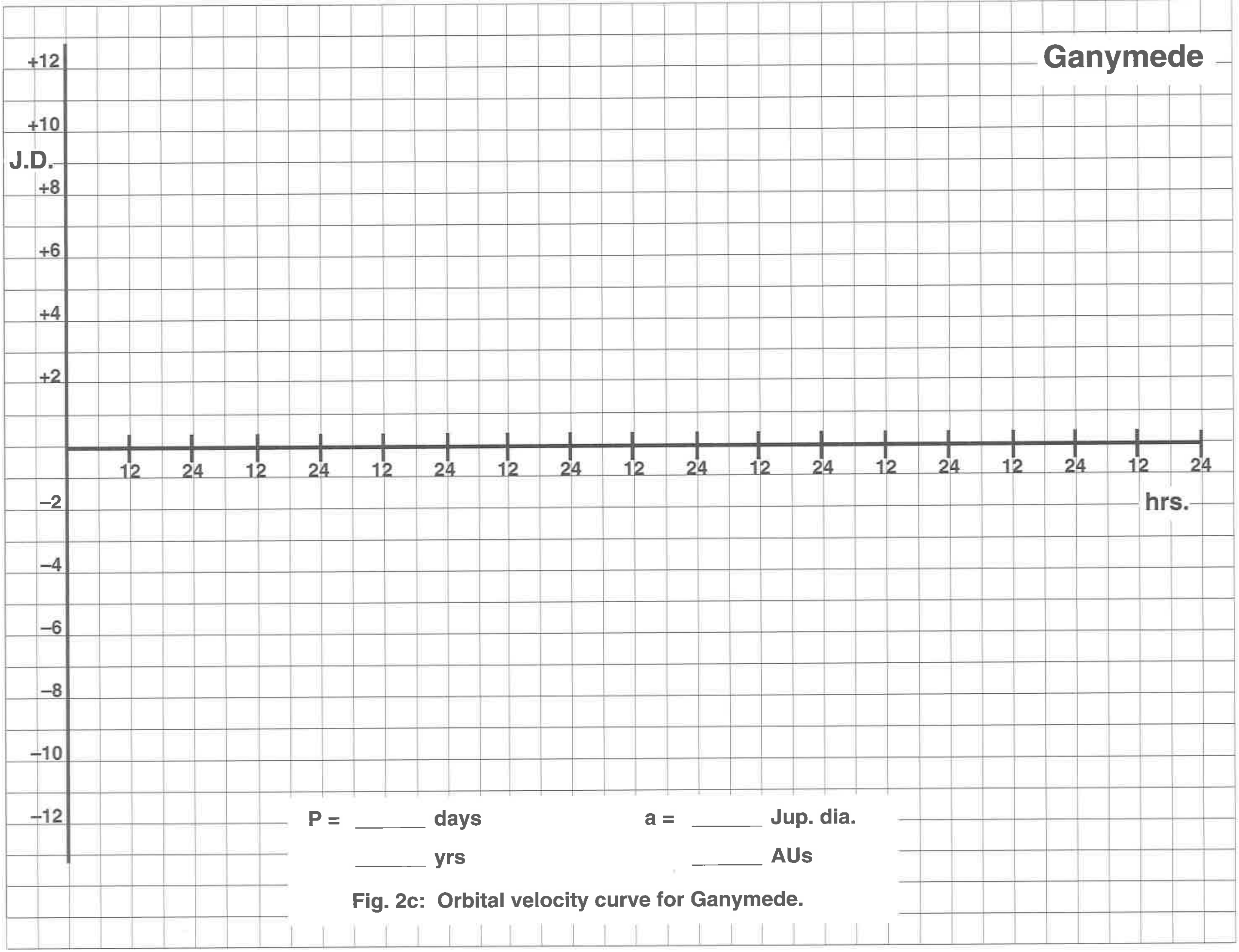
Fig. 2a: Orbital velocity curve for Io.



P = ______ days   a = ______ Jup. dia.

______ yrs   ______ AUs

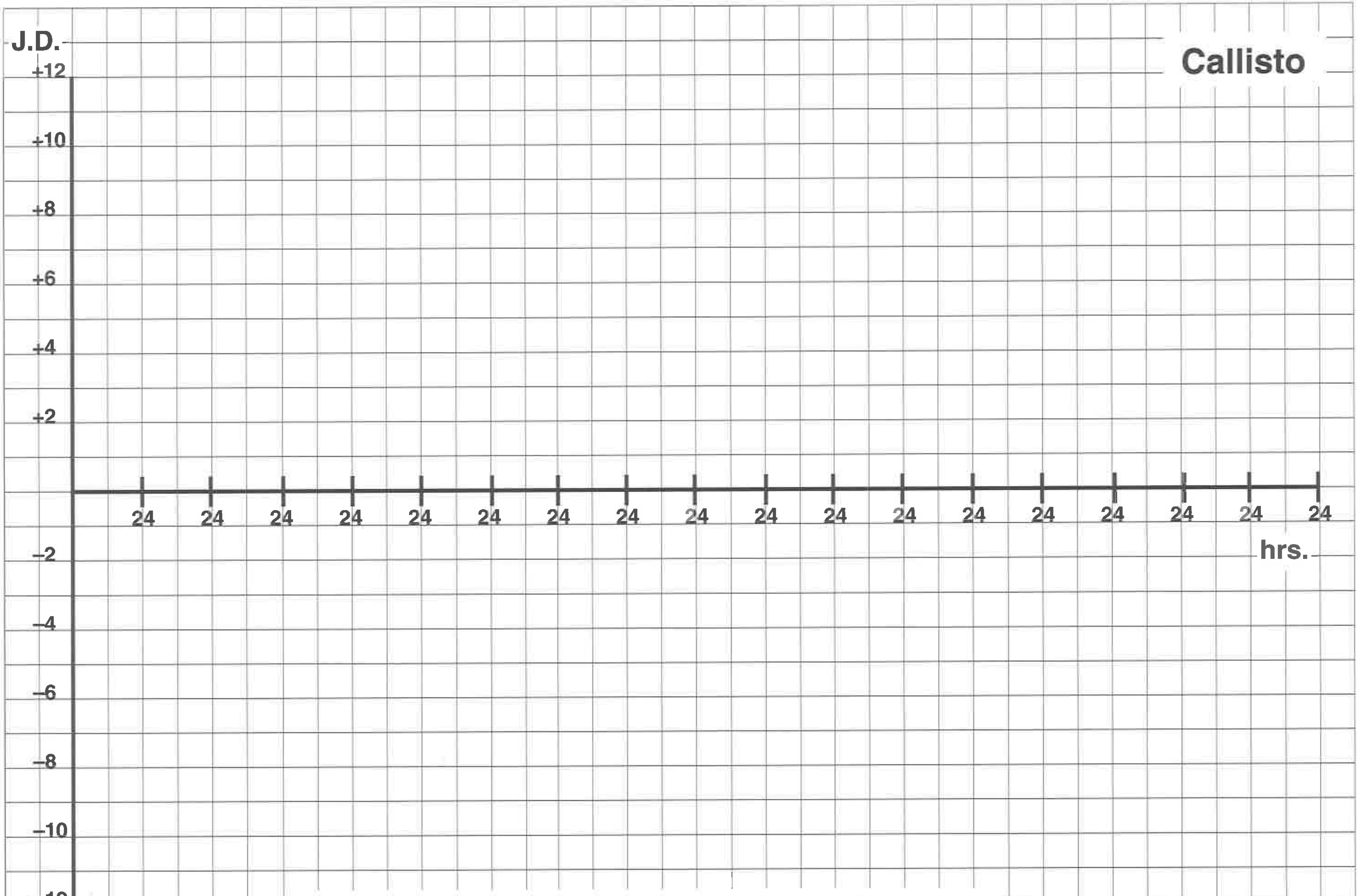
Fig. 2b: Orbital velocity curve for Europa.



P = ______ **days** **a =** ______ **Jup. dia.**

______ **yrs** ______ **AUs**

Fig. 2c: Orbital velocity curve for Ganymede.



P = _______ days

_______ yrs

a = _______ Jup. dia.

_______ AUs

Fig. 2d: Orbital velocity curve for Callisto.

NAME: ____________________

SECTION: ____________________

Questions and Calculations

1. Use **Eq. 3** in the Introduction section of this lab to calculate the mass of Jupiter in terms of the Sun's mass ($M_\odot$). SHOW YOUR WORK.

M_J = ____________________ $M_\odot$

2. The Earth has a mass of 3.0×10^{-6} $M_\odot$. Convert the mass of Jupiter in question 1 into Earth masses ($M_\oplus$) by dividing the mass of Jupiter by the mass of the Earth given above. SHOW YOUR WORK.

M_J = ____________________ $M_\oplus$

3. The accepted mass of Jupiter is about 318 $M_\oplus$. Use the equation below to calculate the percent error in your Jovian mass calculated in question 2. SHOW YOUR WORK.

$$\%\ \text{error} = \frac{\text{accepted value} - \text{your value}}{\text{accepted value}} \times 100$$

% error = ____________________ %

4. The smallest known stars have a mass of about 10% of the Sun's mass (0.1 $M_\odot$). Based on your Jovian mass calculated in question 1, how many times more massive does Jupiter need to be in order to become a small star?

Example 1: A sample calculation of Jupiter's mass based on data in Figure 1.

$$p^2 (M_1 + M_2) = a^3$$

$$M_1 + M_2 = \frac{a^3}{P^2} = \frac{(2.482 \times 10^{-3})^3}{(4.900 \times 10^{-3})^2}$$

$$= \frac{15.29 \times 10^{-9}}{24.01 \times 10^{-6}}$$

$$= 0.6368 \times 10^{-9-(-6)}$$

$$= 0.6368 \times 10^{-3}\ M_{\odot}$$

$$= 6.4 \times 10^{-4}\ M_{\odot}$$