ECLIPSING AND SPECTROSCOPIC BINARY STARS

12

OBJECTIVES

After completing this exercise the student will be able to:

1. determine graphically the observed contact time for primary eclipse of an eclipsing binary.
2. determine the spectroscopic and photometric orbital elements of a double-line spectroscopic eclipsing binary star.
3. calculate the masses and radii of the two stars in the binary system from the orbital elements.

STUDENT MATERIALS

pencil
ruler
calculator

LAB MATERIALS

photometric and spectroscopic data for several eclipsing binaries (optional)

STUDENT REQUIREMENTS

This lab is to be done individually, without lab partners. After completion, turn in answer sheet, **Fig. 3**, and **Fig. 4**.

INTRODUCTION

When a binary star's orbit is nearly edge-on to our line of sight, the two stars will periodically pass in front of each other and eclipse as shown in **Fig. 1a**. It can be seen that two eclipses occur during the complete orbital cycle. The deeper of these two eclipses is called primary eclipse and occurs when the hotter star is eclipsed by the cooler star. Secondary eclipse is the shallower of the two eclipses and occurs when the cooler star is eclipsed by the hotter star. Note that these eclipses are designated according to the relative temperatures of the two stars and not according to their relative sizes or brightness. When the stars are not eclipsing, the light from both stars is observed and causes the constant light level seen during the out-of-eclipse phases of the light curve.

Some eclipsing binaries are also spectroscopic binaries, and their orbital motion can be observed as a periodic Doppler shift of their spectral lines. Single-line spectroscopic binaries show only the spectral lines of the brighter star. The spectral lines of the fainter star are not visible because they have been flooded out by the intense light of the brighter star. Double-line spectroscopic binaries show spectral lines from both stars in the system. This can only occur if the two stars of the system have about the same brightness. Usually one star will be no more than about twice as bright as its companion. **Fig. 1b** shows a hypothetical double-line spectroscopic binary which has circular orbits along with the resulting radial-velocity curves. The measured radial-velocity is composed of two parts; the center-of-mass radial velocity is usually called the γ-velocity and occurs where the two curves intersect. In **Fig. 1b** the γ-velocity of the system is +25 km/sec. A positive radial velocity indicates that the system is moving away from the Sun and a negative radial velocity indicates that the system is approaching the Sun.

It can be seen that the light curve in **Fig. 1a** and the radial-velocity curve in **Fig. 1b** are interrelated. At positions 1 and 3 both stars are moving across our line of sight. At these positions neither star has a radial-velocity component which is produced by the orbital motion of the stars. The light curve shows that either primary or secondary eclipses must be occurring. At position 2, Star A has the maximum possible orbital component of its radial velocity away from the Sun and Star B has the maximum possible orbital component of its radial velocity toward the Sun. At position 4, these conditions are reversed. From the light curve it can be seen that near positions 2 and 4, the full amount of light from both stars is seen and thus the light curves are flat and show the maximum light of the system.

Now let's examine in detail a hypothetical central eclipse of two spherical stars. **Fig. 2** shows the light curve of such a system during primary eclipse and the relative positions of the two stars as the cooler star passes in front of the hotter star. At position A, just before the eclipse begins, the light from both stars is seen to be constant and is defined to be equal to 1 (100%). First contact occurs at position B when the light level begins to fade as the hotter star is eclipsed. At position C, second contact occurs when the hotter star becomes totally eclipsed. The light level remains constant until third contact is reached at position E. It should be noted that between second and third contact only light from the cooler star can be observed since the hotter star is completely hidden from view. From third contact at position E until fourth contact at position F the light steadily increases until it is again equal to 1 (one) at position F, when the light from both stars is observed once again. The two stars have now reversed their original orientation from that seen before first contact.

PROCEDURE

I. Photometric Elements of the Totally Eclipsing Binary SS Bootis

During the spring of 1980 the RS Canum Venaticorum type eclipsing binary SS Bootis (SS Boo) was observed photoelectrically using the 24-inch Seyfert telescope at Dyer Observatory of Vanderbilt University and the No. 4 16-inch telescope of Kitt Peak National Observatory near Tucson, Arizona. The eclipse data obtained are shown in **Fig. 3**. (The data shown have been corrected for the effects of star spots, reflection, and ellipticity.) The time axis (horizontal) is calibrated in fractional Julian Day units. The Julian Day (H.J.D.) is the number of days in a consecutive count starting at noon on January 1, 4713 B.C., as described in the April 1981 issue of ***Sky and Telescope***. The light-level axis (vertical) is calibrated so that the light out-of-eclipse, when the light from both stars is seen, is equal to 1 or 100%. Thus the light seen throughout the eclipse is expressed as a fractional part of the total light of the system or as a percent of the system's total brightness. For this exercise we shall assume the stars in the system are spherical and in circular orbits.

A. Relative Luminosity

The brightness, or luminosity, of each star relative to its companion can be determined from the light level along the bottom of the eclipse. From spectroscopic data it has been determined that the cooler star is a K-type subgiant and the hotter star is a G-type main-sequence star. Since the primary eclipse is the eclipse of the hotter star by the cooler star, and since the cooler star is the larger, it can be seen from **Fig. 2** that all the light observed during totality must originate only from the cooler star. Therefore, the relative luminosity, L_C, of the cooler star in the SS Boo system can be directly determined by extending the eclipse bottom in **Fig. 3** toward the left until it intersects the light-level axis. Read off the light level of this intersection to determine the value of L_C. Record your answer in the space provided on the answer sheet. Recall that the total light of the system, outside eclipse when both stars are fully visible, was defined to be equal to 1 (one). Therefore the relative luminosity L_h of the hotter star must be given by

$$L_h = 1 - L_C \qquad (1)$$

From your value of L_C and equation **(1)** calculate the value of L_h. Record your answer in the space provided.

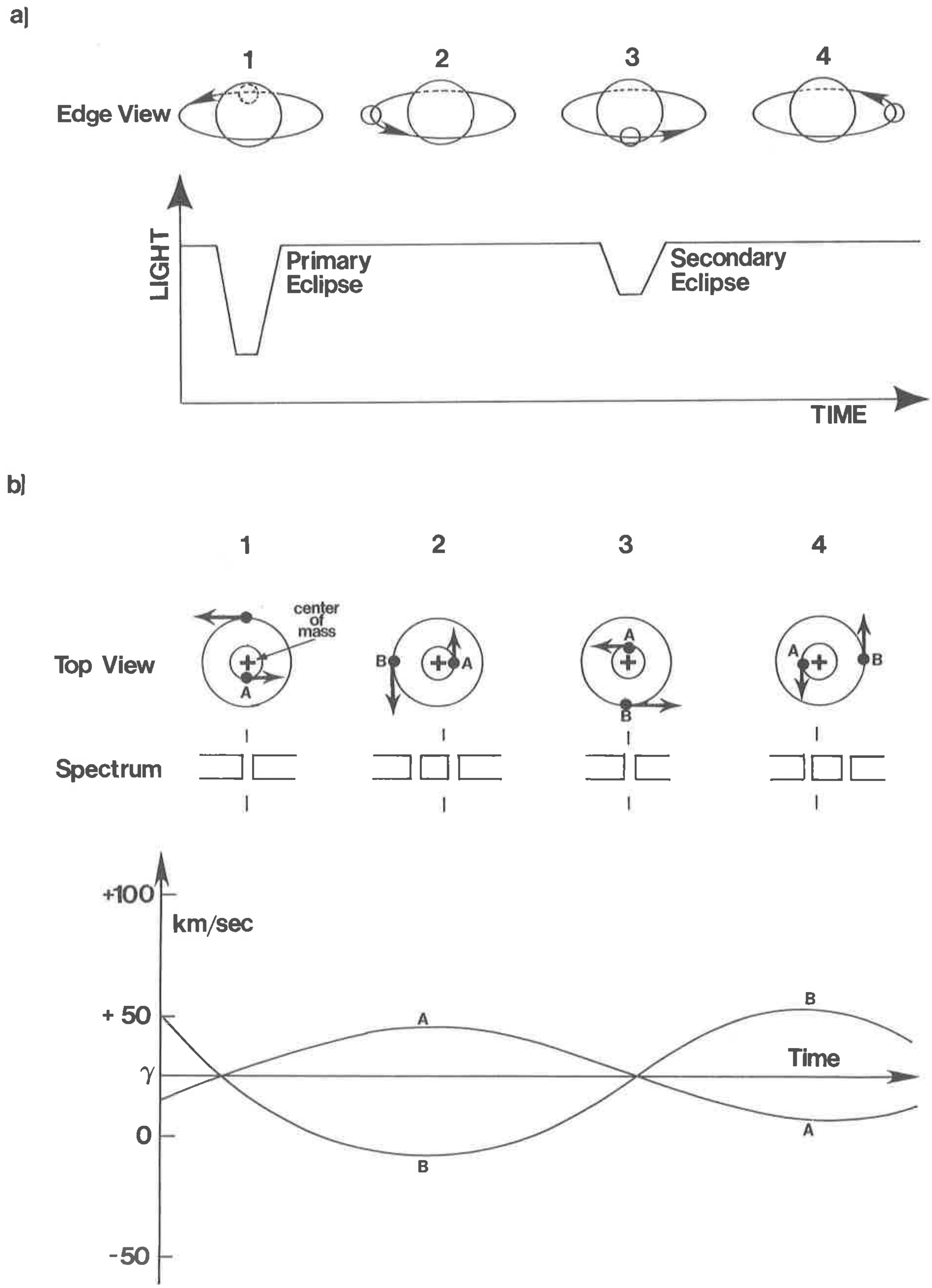

Fig. 1: Comparison of photometric **(a)** and spectroscopic **(b)** data for a hypothetical eclipsing binary with spherical stars and circular orbits.

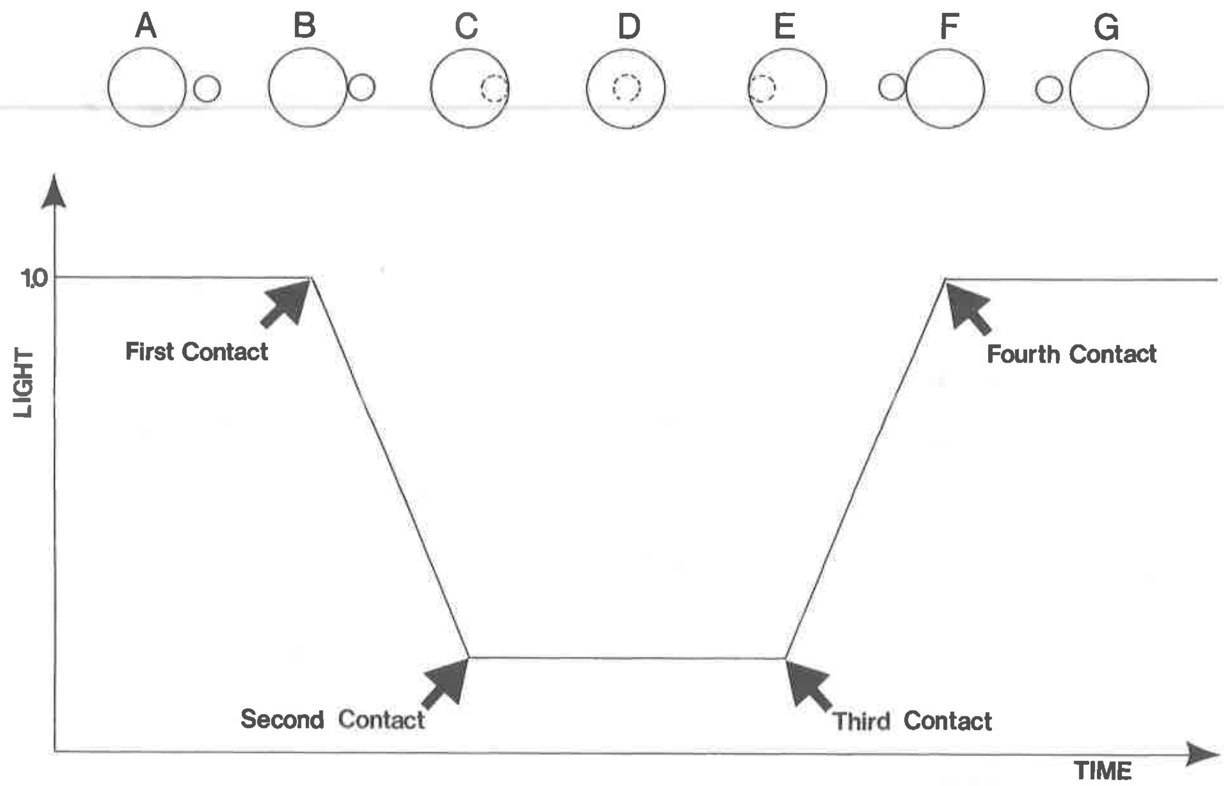

Fig. 2: An expanded view of primary eclipse for the hypothetical binary in **Fig. 1**.

B. Relative Stellar Radii

The location of the first, second, third and fourth contacts was described in the last part of the introduction. With the aid of this description locate the first, second, and third contacts of the SS Boo eclipse shown in **Fig. 3**. Fourth contact was not observed and will not be used for any orbital calculations. From each of these three contacts drop a vertical line to the Julian-Day axis and read off the time each contact occurred. Write your answer in the space on the answer sheet. Be careful and do not forget about the 2444000+ on the H.J.D.

These contact times can be used to determine the radii of the two stars relative to the orbit's circumference. From **Fig. 2** it can be seen that during the interval of time between the first and second contact the smaller star has moved a distance equal to its own diameter. Similarly, during the time interval between first and third contacts the smaller star has moved a distance equal to the diameter of the larger star. The ratio of these time intervals to the orbital period is the same as the ratio of each star's diameter to the circumference of its orbit. Therefore the relative radii of the cooler star, r_c, and the hotter star, r_h, can be calculated from

$$\mathbf{r_c} = \frac{\pi(\mathbf{t_3} - \mathbf{t_1})}{P} \qquad (2)$$

$$\mathbf{r_h} = \frac{\pi(\mathbf{t_2} - \mathbf{t_1})}{P} \qquad (3)$$

where **P** = orbital period = 7.60614 days

t_1 = time of first contact

t_2 = time of second contact

t_3 = time of third contact

π = 3.1416

With the above value for the period and the previously determined contact times, use equations **(2)** and **(3)** to calculate the relative radii of the two stars in the SS Boo system. Is the hotter or cooler star the larger star in this system?

II. Spectroscopic Elements for SS Bootis

SS Boo is also a double-line spectroscopic binary as described in the introduction and as shown in **Fig. 1b**. From 1934 to 1945, Roscoe Sanford observed SS Boo spectroscopically at Mount Wilson Observatory. His data and a radial-velocity curve for SS Boo are shown in **Fig. 4**. The scatter in these data is caused by the faintness of the star which is about 9th magnitude. The vertical axis represents the radial-velocity of the system in km/sec and the horizontal axis represents the orbital phase, or the fractional orbital position. The phases begin with 0 (zero) at mid-primary eclipse. Thus one-quarter of the way around the orbit corresponds to 0.25 phase units, halfway around the orbit corresponds to 0.5 phase units, and three-quarters of the way around corresponds to 0.75 phase units.

A. Center of Mass Velocity

The radial velocity of the center of mass for SS Boo can be determined by drawing a line which connects the intersections of the two curves in **Fig. 4**. Determine the γ-velocity for SS Boo. Record your answer on the answer sheet.

B. Mass Ratio

The mass ratio of the two stars in the SS Boo system can be determined by measuring the semi-amplitudes of the radial-velocity curves in **Fig. 4**. At phase 0.25 and 0.75 draw a vertical line from the top curve to the bottom curve. Measure in units of km/sec from the center-of-mass radial-velocity line to the top of each curve at phases 0.25 and 0.75, and record these lengths as α_c and α_h respectively. Repeat the above procedure, except this time measure down to the bottom of the curves and record as β_h and β_c respectively. These measurements give us a method of determining the mass ratio of the two stars but not their actual masses. With equation **(4)** below and your measured values of α and β, determine the mass ratio for the stars in SS Boo.

$$\textit{Mass Ratio} = \frac{M_h}{M_C} = \frac{\alpha_C + \beta_C}{\alpha_h + \beta_h} \qquad (4)$$

III. Dimensions of the SS Bootis System

By combining both the spectroscopic velocities with the photometric properties, the actual dimensions of an eclipsing binary such as SS Boo can be calculated. Thus it will be possible to calculate the absolute masses, in solar mass units, for each star in the system, and to calculate the absolute radius of each star in solar radius units.

A. Stellar Masses

Since SS Boo is a totally eclipsing binary it can be assumed that the orbital inclination is 90° (edge-on). Thus, the separation, ***a***, between the two stars can be calculated from

$$a = \frac{(0.211\ V)(P/365.25)}{2\pi} \qquad (5)$$

where **V** is the maximum relative velocity of the two stars in km/sec and where **P** is the orbital period in days. The factor 0.211 converts the velocity from units of km/sec to units of AUs/year. It should be obvious that the factor 365.25 converts the orbital period from units of days into years. Therefore, ***a*** will be in astronomical units. From your previously determined values of α_h and β_c, calculate **V** from the relation

$$V = \alpha_h + \beta_c \qquad (6)$$

Use this value of **V** and the known value of **P** to calculate the stellar separation, ***a***, using equation **5**. Record your answer on the answer sheet. The sum of the stellar masses can now be calculated from the values of ***a*** and **P** using Kepler's Third Law, which states

$$M_h + M_c = \frac{a^3}{(P/365.25)^2} \qquad (7)$$

Use the mass sum and the mass ratio previously determined to estimate the individual masses of each star in the SS Boo system. Record these values on the answer sheet.

B. Stellar Radii

We can now calculate the absolute radius for each star in terms of the Sun's radius with the equations

$$R_c = r_c\, a(214.95) \qquad (8)$$

and

$$R_h = r_h\, a(214.95) \qquad (9)$$

where r_c, r_h, and ***a*** have previously been determined.

The factor 214.95 is the number of solar radii in one astronomical unit and converts the radii obtained from astronomical units to solar radius units. Calculate the values $\mathbf{R}_c$ and $\mathbf{R}_h$ for SS Boo. Record the results on the answer page. Stop and think about your answers. Do they make sense? Recall that one star is a K-type subgiant and the other star is a G-type main-sequence star. The above procedure is one of the few methods by which astronomers can obtain information about the masses and sizes of stars. Thus it is very important to modern astronomy.

NAME: ____________________

SECTION: ____________________

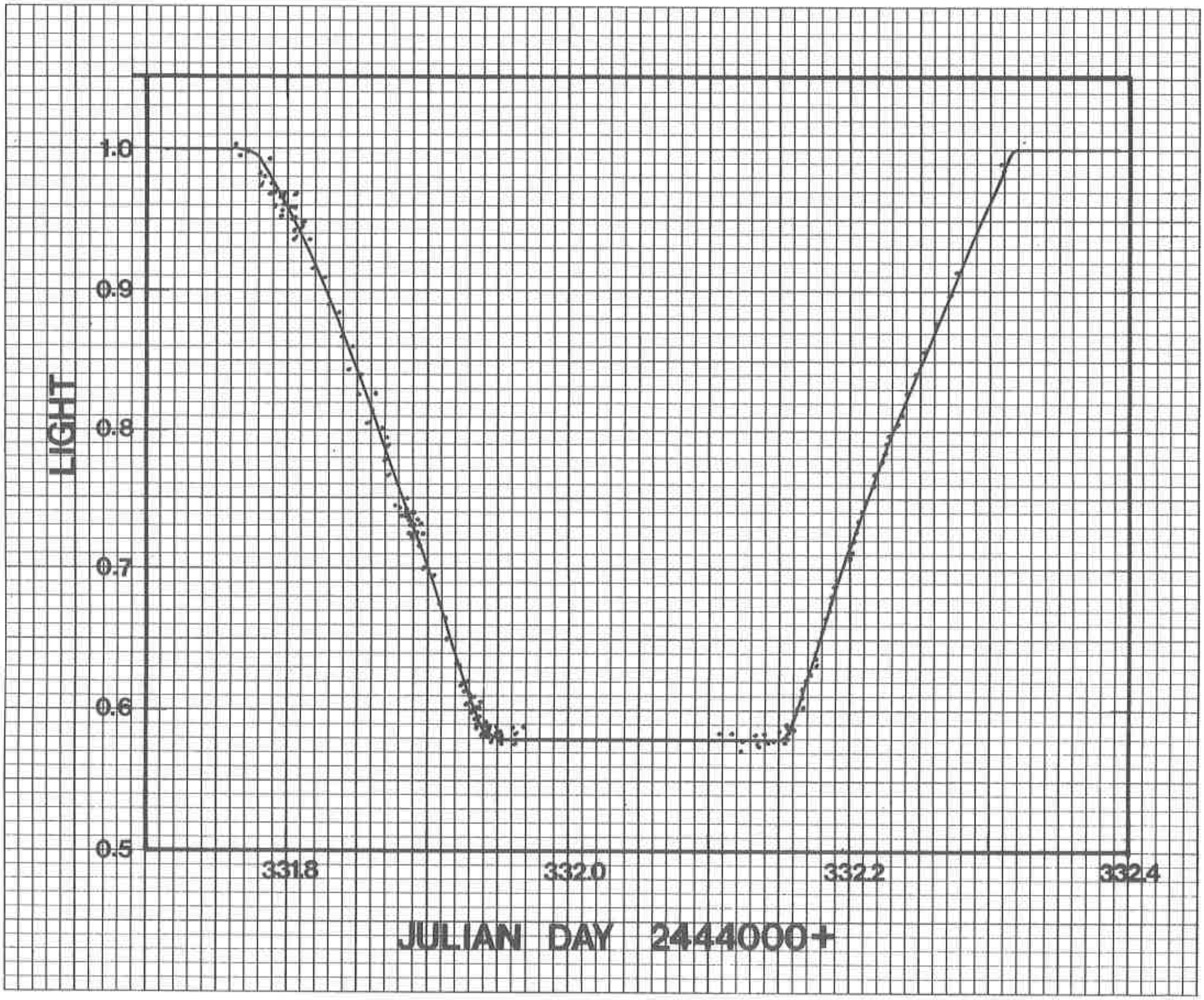

Fig. 3: Primary eclipses of SS Boo.

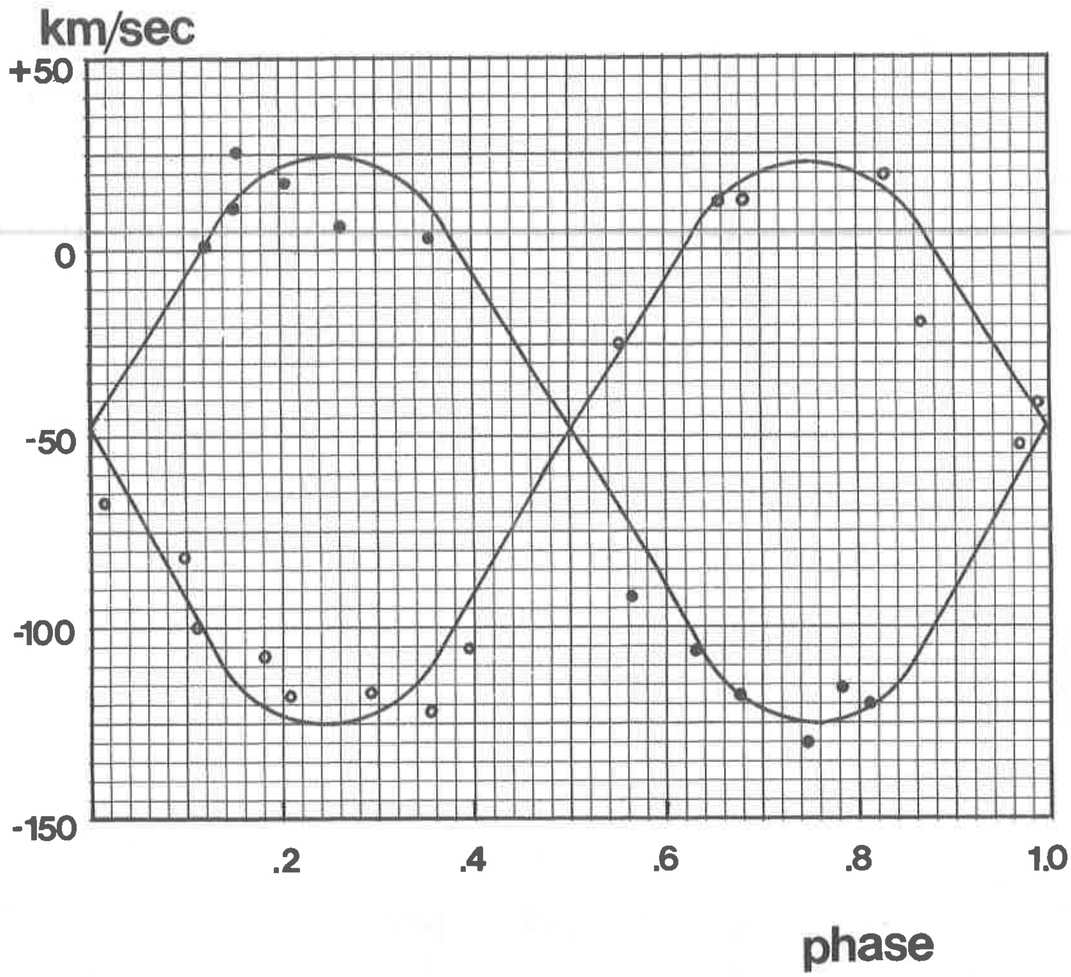

Fig. 4: Radial-velocity curve for SS Boo. Filled circles represent cooler star and open circles represent hotter star.

NAME: ______________________________

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Eclipsing and Spectroscopic Binary Stars

I. Photometric Elements for SS Bootis

A. Relative Luminosity

L_c = ______________ L_h = ______________

B. Relative Stellar Radii

J.D.(hel.) of 1st contact = t_1 = ______________

J.D.(hel.) of 2nd contact = t_2 = ______________

J.D.(hel.) of 3rd contact = t_3 = ______________

r_c = ______________ r_h = ______________

II. Spectroscopic Elements for SS Bootis

A. Center of Mass Velocity

γ = ______________ **km/sec**

B. Mass Ratio

α_c = ______________ km/sec β_c = ______________ km/sec

α_h = ______________ km/sec β_h = ______________ km/sec

Mass ratio = ______________

III. Dimensions of the SS Bootis System

A. Stellar Masses

Total relative velocity = V = ______________ km/sec

separation *a* = ______________ AUs

$M_c + M_h$ = ______________ $M_\odot$

M_c = ______________ $M_\odot$ M_h = ______________ $M_\odot$

B. Stellar Radii

R_c = ______________ $R_\odot$ R_h = ______________ $R_\odot$