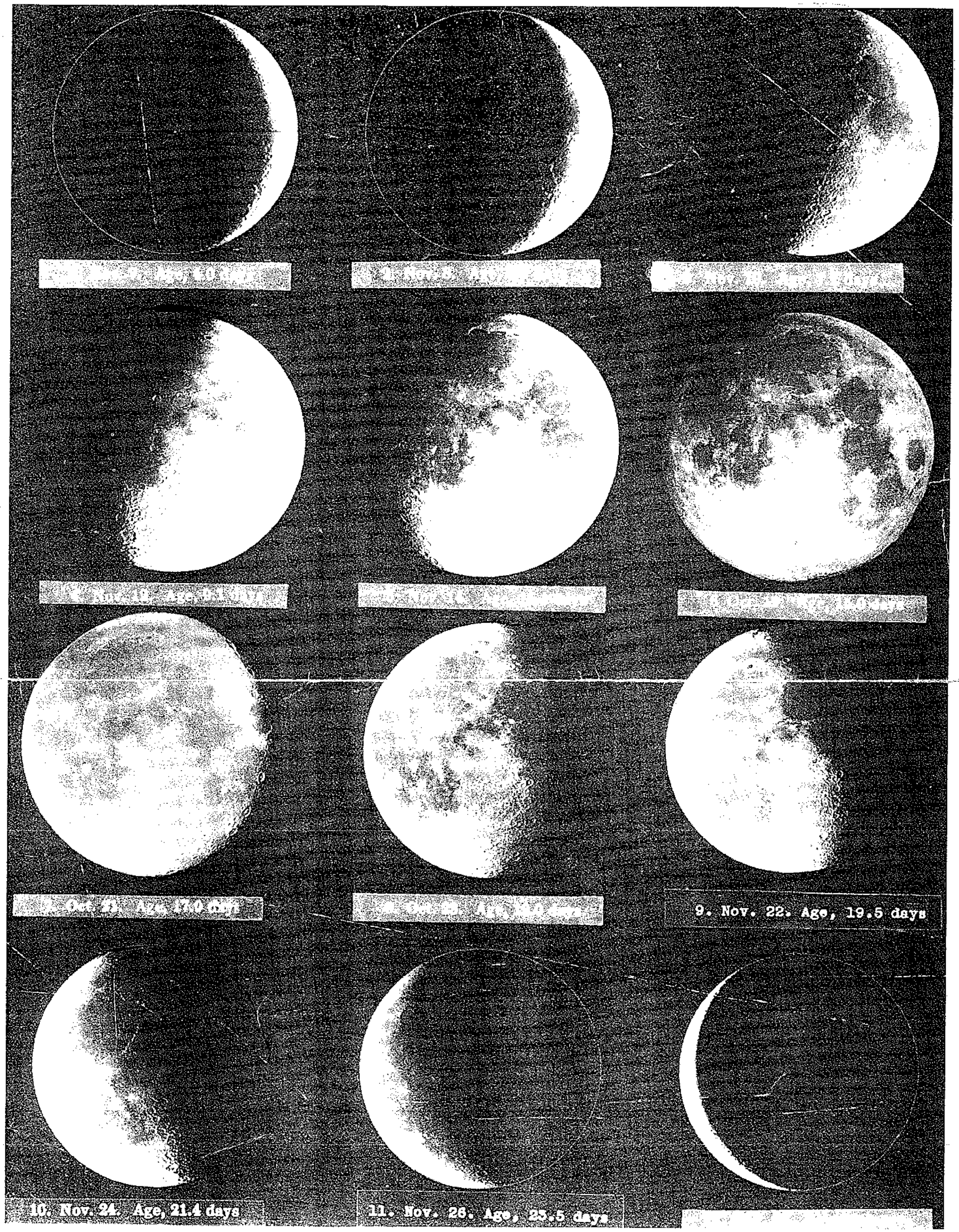

Photographs of the moon taken in October and November, 1918, with the 12-inch refractor of Whitin Observatory, Wellesley College, in Massachusetts. North is shown at the top, as in a naked-eye view, with west to the right. The ages given with each date refer to time elapsed since the preceding new phase, and are not needed in the orbit-plotting experiment. The scale of the students' graphs can be converted to miles by using 225,400 miles as the distance between the centers of the earth and the moon when picture No. 5 was taken, at November 14.5, 1918. At that time the moon's angular diameter was 32′ 58″, and its horizontal parallax was 60′ 24″. Photographs by John C. Duncan.

Laboratory Exercises in Astronomy — The Moon's Orbit

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UNLIKE chemistry and physics, astronomy is primarily an observational rather than an experimental science. Yet direct astronomical observations are subject to the weather and may require working at hours of the night that are difficult for you, as a student, to fit into your schedule.

For example, to see the moon in all its phases for a month requires watching it rise ahead of the sun in predawn skies. The measurement of its diameter at such times can be more conveniently made by means of photographs like those on the facing page.

The time between repetitions of lunar phases, that is, from one new moon to the next, is about 29½ days, the basis for our calendar month. This is the moon's *synodic* period. For experiment, however, we are concerned with the *sidereal* period of about 27⅓ days — one complete revolution of the moon in its orbit with reference to the stars.

John C. Duncan devised this experiment and selected these photographs to represent one sidereal period. Changes in image size correspond to variations in the moon's apparent diameter as its distance from the earth varies, making it possible to determine the form of the moon's orbit relative to the earth.

Details of the lunar motion are exceedingly complex, for as the moon revolves under the influence of the earth's gravitation, it is highly perturbed by the sun. This is a special case of the three-body problem in celestial mechanics.

But to a first approximation, the lunar orbit may be considered an ellipse, and the moon may be regarded as following Kepler's laws. In this project we shall infer an orbit from measurements of the moon's apparent diameter, using a circle not centered on the earth to approximate the elliptical shape of the orbit.

When the moon is closest to the earth (at *perigee*), it appears appreciably larger than when it is farthest (at *apogee*). Even the ancient Greek astronomers knew this. The moon's distance, R, is inversely proportional to its angular diameter, d, that is,

$$R = \text{constant}/d. \qquad (1)$$

Procedure. Carefully measure the diameter of each image, estimating the *tenth of a millimeter;* where possible, use various diameters and form an average. These measurements must be made as precisely as possible, because the entire experiment depends on them. Use a precision scale (engineer's rule) and, perhaps, a hand magnifying glass.

Present your results systematically in a table based on the one at right, adding columns for the measurements, the adopted value of d, and a final column for R.

At this stage the changes in apparent diameter are exhibited in the table on an arbitrary scale. That is, if measured in inches instead of millimeters, the variation would still be there although the numbers would be different. Similarly, changes in the lunar distance, again on an arbitrary scale, may be found from Equation 1. For convenience in plotting, let us choose the constant to be 4,000. Now proceed to fill in the column for the relative distance R from

$$R = 4{,}000/d. \qquad (2)$$

The table gives the date of each picture to decimals of a day. (Equivalent November dates have been assigned to the three October photographs.) The geocentric longitude and latitude of the moon for each date have been taken from tables in the 1918 *American Ephemeris.*

Plotting the Orbit. Mark a point near the center of an unruled sheet of paper eight inches wide, and draw a reference line to the right. Lay out the 12 longitude angles with a protractor, measuring counterclockwise from the reference line. Along each angle measure off the appropriate relative distance in millimeters and mark this position with a conspicuous dot. Each of these directed lengths is a radius vector.

Add up all your values of R and find their average. Cut out a paper disk having this average radius and adjust it to fit the 12 points on the graph as well as possible. With the disk so placed, mark its center on the paper and draw the circle with a compass. This represents the moon's orbit; it is not an ellipse, but a very close approximation.

Major Axis and Foci. The point from which the angles were measured and the radius vectors drawn is the position of the earth, and one focus of the ellipse. Using a colored pencil, draw the major axis through the earth's position and the center of the circle; construct the minor axis perpendicular to the first line. Label the major and minor axes and the foci.

Image	*Date*	*Longitude*	*Latitude*
1	Nov. 7.44	270°	+1°.5
2	8.46	283°	+2°.6
3	10.46	309°	+4°.3
4	12.46	336°	+5°.2
5	14.46	5°	+5°.8
6	15.81	27°	+3°.9
7	17.84	57°	+1°.6
8	19.94	87°	−1°.2
9	22.92	127°	−4°.3
10	24.79	151°	−5°.1
11	26.90	176°	−5°.1
12	29.92	212°	−3°.5

Eccentricity. The distance from the center of an ellipse to a focus is ae, where a is the semimajor axis, e the eccentricity. Measure the distance carefully and calculate e.

Line of Apsides. Label apogee and perigee, which are joined by the line of apsides. Determine longitude and date of perigee. Compare your answer with the perigee date of November 16.6 given in the 1918 *Ephemeris.*

Line of Nodes. From the moon's latitude in the table, determine roughly the positions of the ascending and descending nodes, and join them with a line. Estimate the dates when the moon reached the nodes.

Law of Areas. Kepler's second law requires that the moon's radius vector sweep out equal areas in equal time intervals. The area A between any two of your plotted points can be found with sufficient accuracy from

$$A = r_1 r_2 \pi \theta/360, \qquad (3)$$

where θ is the angle between the radius vectors r_1 and r_2.

The area swept out per unit time is called the *areal velocity,* and can be expressed from the graph in square millimeters per day. Find the areal velocity in the region of perigee, and near apogee.

Tabulate your results, as well as the quantities used for deriving them.

NOTE FOR TEACHERS: This exercise is designed for a 90-minute laboratory period in introductory astronomy. It can be shortened by omitting the law of areas and areal velocity requirements, or by providing a short table of $4{,}000/d$ values.

It is also possible to use a current edition of the *American Ephemeris* and have the students take from it values of the moon's semidiameter for every date during a month, giving more accurate orbits. Longitudes and latitudes can also be treated this way.

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