

OBJECTIVES

After completing this exercise, the student will be able to:

1. draw a simple ellipse using two thumbtacks and a loop of string.
2. calculate the eccentricity of an ellipse.
3. use a series of lunar photographs to make a scale drawing of the moon's orbit.
4. be able to confirm the elliptical nature of the moon's orbit by determining the major axis, minor axis, and eccentricity of the orbit.

STUDENT MATERIALS

pencil
 mm ruler
 compass
 protractor
 calculator
Sky and Telescope reprint, "The Moon's Orbit," from back of lab book

LAB MATERIALS

two thumbtacks per student
 one loop of string per student
 a piece of cardboard (~8 1/2 x 11") per student

STUDENT REQUIREMENTS

This lab is to be done individually without lab partners. After completing the lab, turn in **Table I**, **Fig. 1**, and **Fig. 2** in page order sequence.

PROCEDURE

A. Kepler's Laws and the Elliptical Orbit

You will draw an elliptical orbit on the page provided (**Fig. 1**). To do this you will need a piece of cardboard, two thumbtacks, and a loop of string.

1. Remove **Fig. 1** and place it on the piece of cardboard.
2. Insert a thumbtack through the two focus points labeled **F1** and **F2**.
3. Make a loop of string which is large enough to go around both tacks at the same time. The loop should stretch about 8 to 10 cm, but its actual size is not important.
4. Place the loop of string around the tacks and stretch it taut with a sharp pencil point as demonstrated by your instructor. While holding the pencil vertically and keeping the string taut draw an ellipse by moving the pencil around the two tacks. The orbit you have drawn is similar to that of some comets and asteroids.
5. Draw a line longwise through the ellipse which connects **F1** and **F2**. This line is called the major axis. Label the major axis on your drawing.
6. The minor axis is perpendicular to the major axis and divides it in half. The midpoint of your major axis is halfway between **F1** and **F2**. Locate this point and label it **c**. Now draw and label the minor axis on your ellipse drawing.
7. Kepler's 1st law states that the Sun is at one focus of an elliptical orbit. Pick one of the foci and label it **Sun**. The other focus has no physical significance.
8. Perihelion is the closest approach to the Sun. Aphelion is the farthest point from the Sun. Perihelion and aphelion are on the orbit at each end of the major axis. Find and label these two points on your orbit.
9. The distance from the center **c** to either perihelion or aphelion is named the semi-major axis. Label this distance as **a** on your orbit. The distance from the center to one of the foci is named **x** in this exercise. Label this distance on your orbit.

10. On your orbit, measure, in AUs (1 cm = 1 AU) the perihelion distance, the aphelion distance, and the major axis, the semi-major axis, and the distance, x . Record your results to a tenth of an AU in the spaces provided below your drawing.
11. How much an ellipse deviates from a circle is called its eccentricity, e . Calculate your orbit's eccentricity using the relation $e = x/a$, and record your answer in the space provided. Note that eccentricity is a ratio and has no units.
12. Kepler's 3rd law states that the orbital period, P , is related to the orbit's semi-major axis, a , by the relation $P^2 = a^3$, where P will be in years if a is in AUs. Use the 3rd law to calculate the period of your orbit. Record your answer to the nearest tenth of a year in the space provided.

B. The Moon's Orbit

As applied to the moon's orbit, Kepler's 1st law would state that the moon's orbit is an ellipse with the Earth at one focus. If the moon's orbit is an ellipse its distance from the Earth should change during one complete orbit. This means the moon should appear larger at perigee (closest approach to Earth) than it does at apogee (farthest from Earth). The *Sky and Telescope* reprint provided by your instructor shows a series of lunar photographs. These were obtained as the moon passed several different positions in its orbit. It is possible to use these photographs to determine a few orbital properties of the moon.

1. Measure the diameter, d , of each lunar image shown. Be careful not to simply measure the illuminated portion. Make all measurements more or less vertically. Estimate each value to a **tenth** of a **millimeter** using a magnifier. Record your measurements in the column provided in **Table I**. (**Note:** these values should range between 40 mm and 60 mm.)

2. From each diameter measurement the relative distance to the moon can be determined. This distance will be scaled down to fit on a piece of graph paper. Calculate the scale distance, D , (in mm) to the moon using the relation $D = 4000/d$, where d is the diameter recorded in column 4 of **Table I** and 4000 is a scale factor. Record your values of D , to its nearest tenth of a millimeter, in the last column of **Table I**.
3. Calculate the average scale distance, D_{ave} , to the moon and record your answer in the space provided at the bottom of **Table I**.
4. Plot the moon's orbital position on the polar coordinate graph paper given as **Fig. 2**. You will have to plot longitude vs. scale distance, D . On the graph paper longitude starts at the bottom, labeled 0° , and increases counterclockwise. This will correspond to the true direction of the moon's orbital motion as viewed from above the Earth's north pole.

To plot the first data point, place the edge of your millimeter ruler along the line labeled 270° (longitude). Be sure that 0 mm is at the origin of the graph paper, which corresponds to the Earth's position on this diagram. Now measure out along the 270° line and place a pencil dot at your value of D for a longitude of 270° . Continue in this manner for the remaining data in **Table I**.

5. If you examine the plotted data on **Fig. 2** it can be seen that they are **not** centered around the Earth. In order to draw the moon's orbit you must first establish the orbit's center. Open your compass equal to the D_{ave} calculated in step 3. Remember to use the edge of a ruler to properly set the compass. Locate the center by placing the compass point on three or four of the plotted data points and draw a small arc near the center of the graph. Ideally these should all cross at one place, the center. In reality, they will show you only approximately where the center should be.

6. Place the compass point within the small area enclosed by your arcs drawn in part 5. Check to see how well the compass pencil passes through the data points. This will not be perfect. Adjust the compass point's location until a best-fit circle can be drawn. Once you have located the center, draw in the moon's orbit with your compass. You are thus representing the elliptical orbit of the moon with an offset circle instead of an ellipse.
7. The compass hole is the center of the moon's orbit. One focus is located at Earth, which is at the origin of the graph paper. Use these two points to draw and label the major and minor axes of the moon's orbit. Hint: How are the center and foci related to the major and minor axes in **Fig. 1**?
8. Label perigee (closest) and apogee (farthest) on the moon's orbit. These may, or may not, be at data point locations.
9. Measure the values of x and a in millimeters. Calculate the eccentricity of the orbit using $e = x/a$. Record your results in the spaces provided on the graph paper.

Turn in **Fig. 1**, **Table I**, and **Fig. 2**.

NAME: _____

SECTION: _____

•
F2

•
F1

Perihelion = _____ P = _____

x = _____

a = _____

e = _____

Aphelion = _____

Major Axis = _____

Fig. 1: Drawing of an elliptical orbit.

Kepler's Laws of Planetary Motion

Written by Dr. John Wilson

Introduction

Open the app <http://astro.unl.edu/classaction/animations/renaissance/kepler.html> produced by University of Nebraska Lincoln. This app is a simulation that shows how Kepler's Law acutely describe planetary motion. These laws are so well developed that they can be applied to orbits of moon's orbiting planets such as Jupiter, binary star systems, and to the orbits of extrasolar planets around other stars.

Law 1: Law of Elliptical Orbits

Kepler's 1st Law states that '*planets orbit the Sun in an elliptical path with the Sun at one focus*'. To see how this works, **click 'start animation' button and then click on the tab labeled 'Kepler's 1st Law'**. To relate this to the ellipse figure you drew previously click on show radial lines. Now **click on show empty focus, show center, and show semimajor axis**. (You can click the semi minor axis if you want, but it is not necessary, and sort of gets in the way as clutter.) Adjust the orbit's eccentricity (e) by making e get larger and smaller.

1. Describe what happens to the shape of the ellipse, when *eccentricity* gets larger or smaller?

2. *Change the value of eccentricity* until the orbits becomes circular in shape. What is the eccentricity of a circular orbit?

Name:
Lab Section:

Be sure that the empty focus, center and semi-major axis boxes are still turned on and then **click “set parameters for”** in the upper-right corner to select each planet in the Solar System and **click OK. Complete the second (eccentricity) and third (semi-major axis) columns in Table 1 below.** You will need to *click each planet one at a time* to view their orbits. *Note that the orbital periods will be calculated later in the lab activity.*

Table 1: Orbital parameters of the planets in the Solar System

Planet	Eccentricity (e)	Semi-major Axis (a) [AU]	Orbital Period (P) [years]
Mercury			
Venus			
Earth			
Mars			
Jupiter			
Saturn			
Uranus			
Neptune			
Pluto			

3. Which planet has the most circular orbit? Which planet has the most elliptical orbit?

Name:
Lab Section:

During Kepler's lifetime (in early 17th-century) only the planets Mercury, Venus, Earth, Mars, Jupiter and Saturn were known. Tycho Brahe told Kepler that the data for the positions of Mars were the most difficult to match using a circular orbit. Based on this information Kepler selected the orbit of Mars as the starting point to developing his 1st law of planetary motion.

4. Briefly discuss why the orbit of Mars was so difficult to reconcile with a circular orbit?

Before proceeding to the next part, click the **'reset' button** near the top right corner of the app to refresh the simulation display.

Law 2: Law of Equal Areas

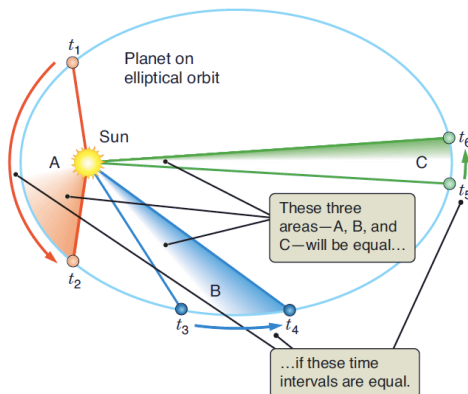


Image: 21st Century Astronomy 4th Edition

Kepler's 2nd law states that *'a line joining the Sun and a planet sweeps out equal areas in equal times'*. Basically this law describes how a planet's speed changes as it orbits the Sun in an elliptical path.

In the lower left corner of the app **click open Kepler's 2nd Law**. Notice that you can control how long a sweep last by adjusting the fractional sweep bar. The default value is a sweep for 1/16 of the orbital period. Now **click 'start animation'** and observe how the speed of the planet changes.

5. Describe how the orbital speed of the planet changes: A) when it is near *aphelion* (which is the farthest point from the Sun), and B) when it is near *perihelion* (which is the closest point to the Sun).

Name:
Lab Section:

As you observe the planet approaching *perihelion* **click the sweep button** and the simulation will show you the area swept out by the line connecting the Sun to the planet (1/16 of the orbital period). **Repeat this procedure** when the planet approaches *aphelion*. Now **sweep a random area as the third one**.

6. Describe how the three areas are different. Are these three areas the same or is one larger than others? If their areas are same or different, *why* is that? (Note that colors are not relevant it is only where to show the three different areas were selected.)

Before proceeding to the next part, **click “reset” button** near the top right corner of the app to refresh the simulation display.

Law 3: The Harmonic Law

The 3rd law describes ‘*how long it takes a planet to orbit the Sun is related to the planet’s average distance from the Sun*’. Mathematically it can be stated as $P^2 \sim a^3$ where P is the orbital period in *years* and, a is the length of the semi-major axis in *Astronomical Units (AU)*.¹ So it describes *the relationship between the orbital period and semi-major axis of the planet relative to the Earth’s orbital period (1 year) and semi-major axis (1 AU)*.

7. Use Kepler’s 3rd Law ($P^2 \sim a^3$) to calculate the orbital period of Mercury, and *complete fourth column* in Table 1. *Repeat* this procedure for all the remaining planets.

8. *Halley’s comet* has an orbital period of around 76 *years*. Calculate this comet’s semi-major axis in AUs. *Show your works!*

¹ Mathematically, tilde (\sim) means ‘approximately equal to’.