Binary Stars

ASTR 1020

Name:

Overview

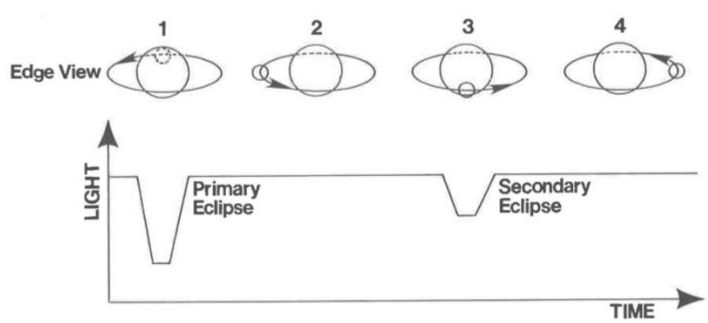
In this activity, you will explore the behavior of two stars in a binary system and use the light curve of the system and the velocities of the two stars to derive physical stellar properties.

Objectives

After completing this activity students will be able to:

- Determine graphically the observed contact time for primary eclipse of an eclipsing binary.
- Determine the spectroscopic and photometric orbital elements of a double-line spectroscopic eclipsing binary star.
- Calculate the masses and radii of the two stars in the binary system from the orbital elements.

Definitions



- **Eclipsing Binary** – a system of 2 stars in which, from the viewpoint of Earth, one star crosses over and eclipses the other star, resulting in a dip in brightness during the eclipse
- **Contact** – (1st contact, 2nd contact, etc.); times in which one star in an eclipsing binary system is eclipsing the other. 1st contact refers to the beginning of an eclipse, 2nd contact refers to the beginning of total eclipse (one star is completely in front of another), 3rd refers to the end of the total eclipse, and 4th refers to the end of the eclipse
- **Orbital Period** – (*P*); the amount of time an eclipsing binary system takes to complete 1 full orbit around each other
- **Phase** – 1 full orbit of an eclipsing binary system
- **Double-Line Spectroscopic Binary** – a binary system in which both stars exhibit absorption lines which, over the course of the orbit, are redshifted to the blueshifted by the relative motions of the stars
- **Center of Mass** – the point in space where two stars in a binary system orbit around

- **Astronomical Unit** – (AU) The average distance between the Earth and the Sun, equal to 1.5×10^{11} m.
- **π** – Equal to 3.14.

Part 1. Photometric Elements of the Totally Eclipsing Binary SS Bootis

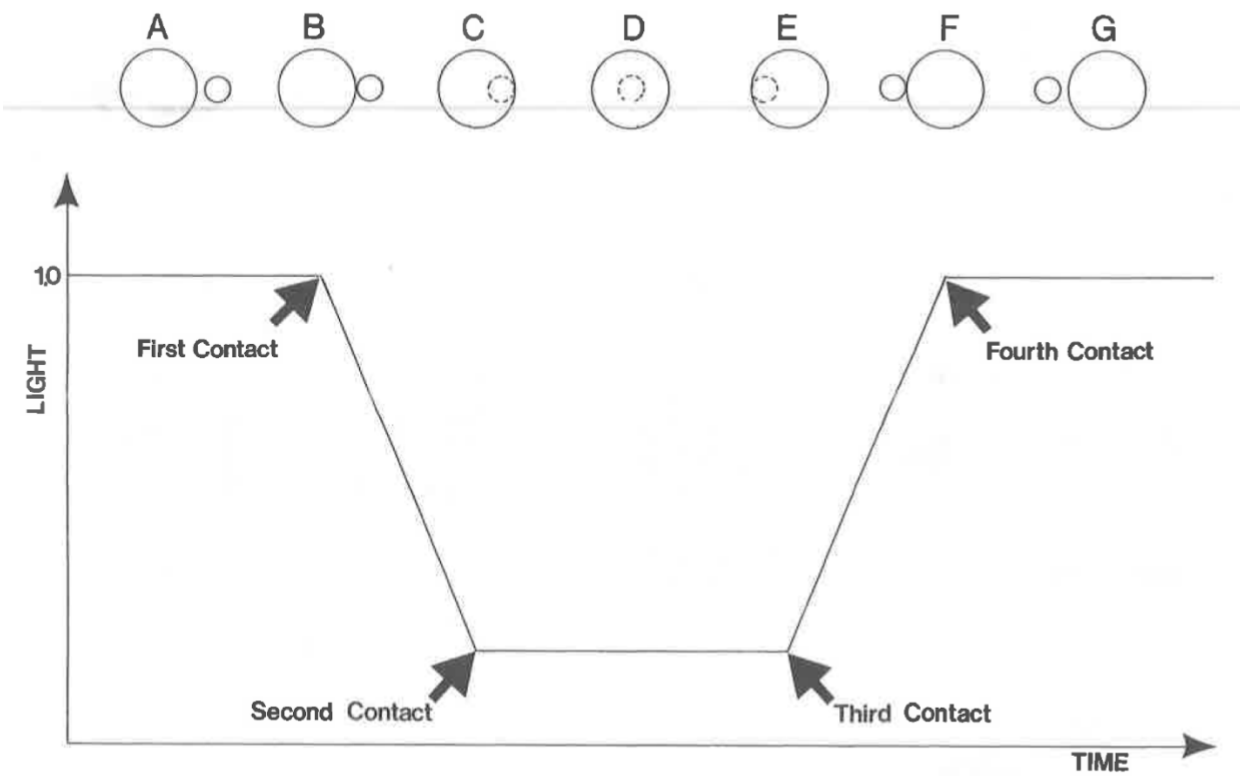


Figure 1: An expanded view of preliminary eclipse for a hypothetical binary

Part A: Relative Luminosity

The brightness, or luminosity, of each star relative to its companion can be determined from the light level along the bottom of the eclipse. From the spectroscopic data it has been determined that the cooler star is a K-type subgiant and the hotter star is a G-type main-sequence star. Since the primary eclipse is the eclipse of the hotter star by the cooler star, and since the cooler star is the larger, it can be seen from Figure 1 that all the light observed during totality must originate only from the cooler star.

Therefore, the relative luminosity of the cooler star (L_c) in the SS Boo system can be directly determined by extending the eclipse bottom in Figure 2 toward the left until it intersects the y-axis. Read off the light level of this intersection to determine the value of L_c. Record your answer below.

1. L_c = ☐

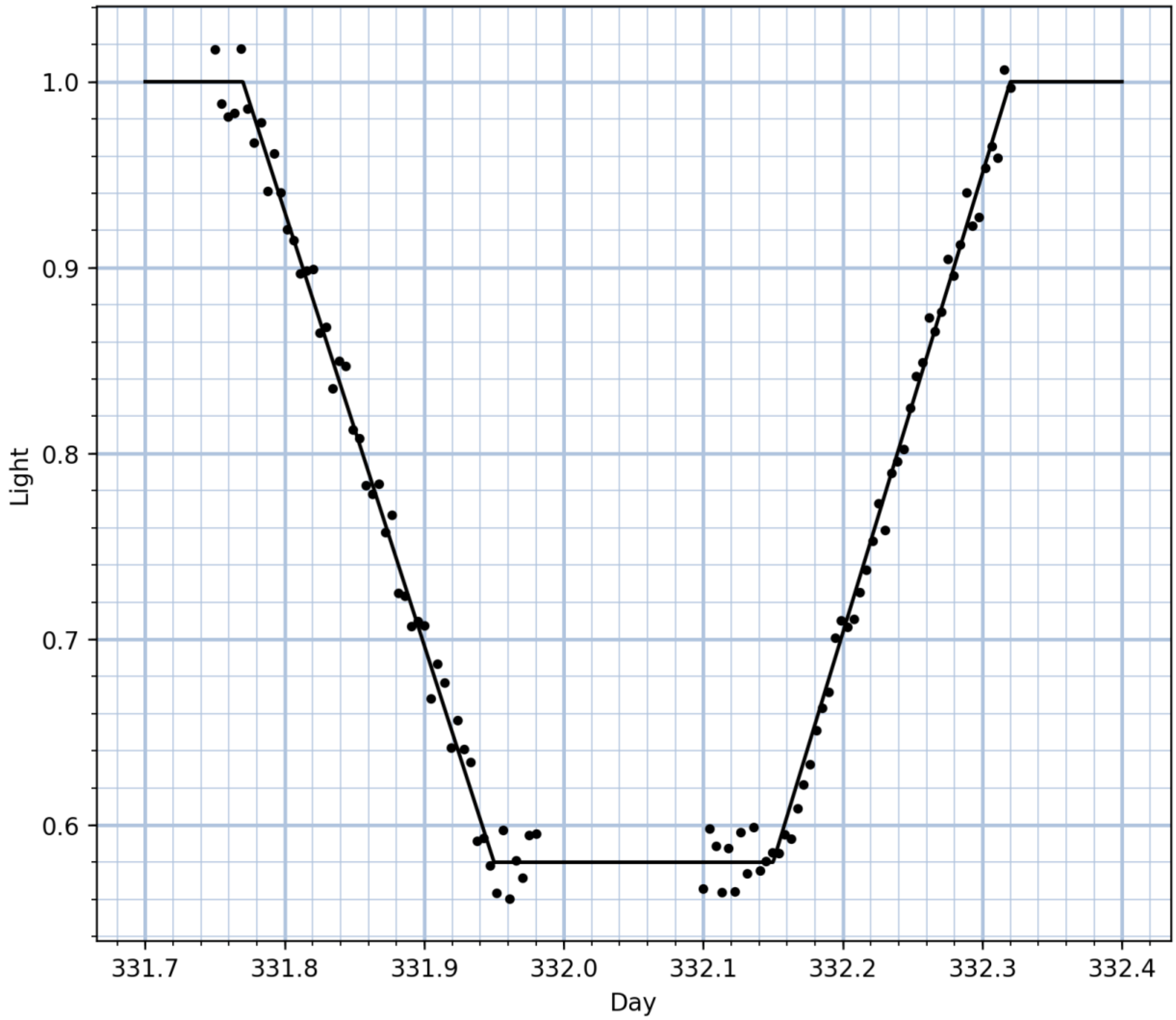


Figure 2: Preliminary eclipses of SS Boo

Recall that the total light of the system, outside eclipse when both stars are fully visible, was defined to be equal to 1. Therefore, the relative luminosity of the hotter star (L_h) must be given by Equation 1:

$$L_h = 1 - L_c \qquad (Equation\ 1) \qquad (1)$$

Use your value of L_c and equation 1 to calculate the value of L_h. Record your answer below.

2. $\boldsymbol{L_h}$ =

Part B: Relative Stellar Radii

The location of the first, second, third, and fourth contacts are labeled in figure 1. Using these, locate the first, second, and third contacts of the SS Boo eclipse shown in figure 2. From each of these three contacts, drop a vertical line to the x-axis and read off the time each contact occurred. Write your answer below.

3. J.D.(hel.) of 1st contact = $\mathbf{t_1}$ = [ ]
4. J.D.(hel.) of 2nd contact = $\mathbf{t_2}$ = [ ]
5. J.D.(hel.) of 3rd contact = $\mathbf{t_3}$ = [ ]

These contact times can be used to determine the radii of the two stars relative to the orbit's circumference. From Figure 1 it can be seen that during the time interval between the first and second contact the smaller star has moved a distance equal to its own diameter. Similarly, during the time interval between first and third contacts, the smaller star has moved a distance equal to the diameter of the larger star. The ratio of these time intervals to the orbital period is the same as the ratio of each star's diameter to the circumference of its orbit. Therefore, the relative radii of the cooler star, r_c, and the hotter star, r_h, can be calculated from Equations 2 and 3:

$$r_c = \frac{\pi(t_3 - t_1)}{P} \qquad (Equation\ 2)$$

$$r_h = \frac{\pi(t_2 - t_1)}{P} \qquad (Equation\ 3)$$

where P = orbital period = 7.60614 days, t_1 is the time of first contact, t_2 is the time of second contact, t_3 is the time of third contact, and π = 3.14. Using the orbital period and your times of contact, use Equations 2 & 3 to calculate the relative radii of the two stars in the SS Boo system.

6. $\boldsymbol{r_c}$ = [ ]
7. $\boldsymbol{r_h}$ = [ ]

Part 2. Spectroscopic Elements for SS Bootis

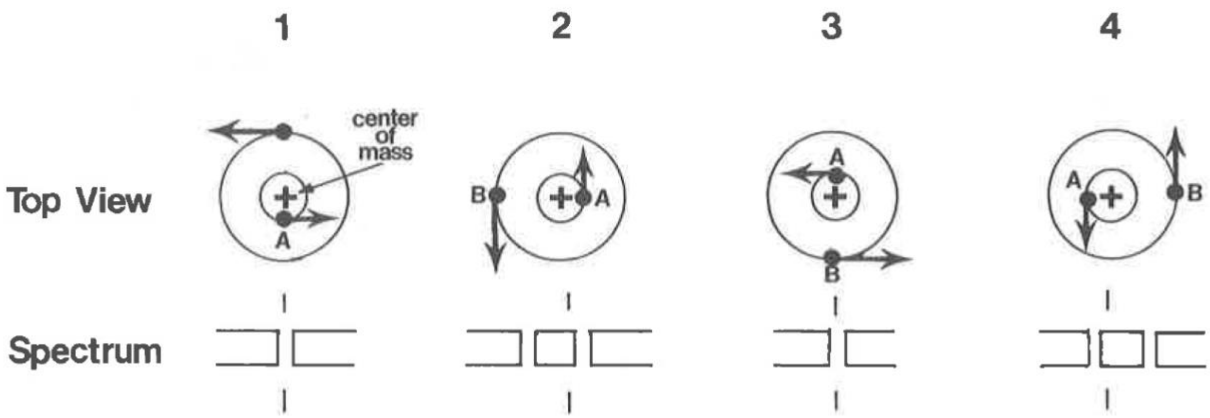


Figure 3: Spectroscopic data for a hypothetical eclipsing binary system

SS Boo is also a double-line spectroscopic binary as shown in figure 3. From 1934-1935, Roscoe Sanford observed SS Boo spectroscopically at Mount Wilson Observatory. His data and a radial velocity curve for SS Boo are shown in figure 4. The scatter in these data is caused by the faintness of the star. The y-axis is the radial velocity of the system in km/s, and the x-axis is the orbital phase, or the fractional orbital position. These phases begin with 0 at mid-primary eclipse. Thus ¼ of the way around the orbit corresponds to 0.25 phase units, and ¾ of the way around corresponds to 0.75 phase units.

Part A: Center of Mass Velocity

The radial velocity of the center of mass for SS Boo can be determined by drawing a line which connects the intersections of the two curves in figure 4. Determine the γ-velocity for SS Boo and record you answer below

8. **γ =** [ ] km/s

Part B: Mass Ratio

The mass ratio of the two stars in the SS Boo system can be determined by measuring the semi-amplitudes of the radial velocity curves in Figure 4. At phase 0.25 and 0.75, draw a vertical line from the top curve to the bottom curve. Measure in units of km/s from the γ-line to the top of each curve at phases 0.25 and 0.75, and record these lengths as α_c and α_h

9. α_c = [ ] km/s

10. α_h = [ ] km/s

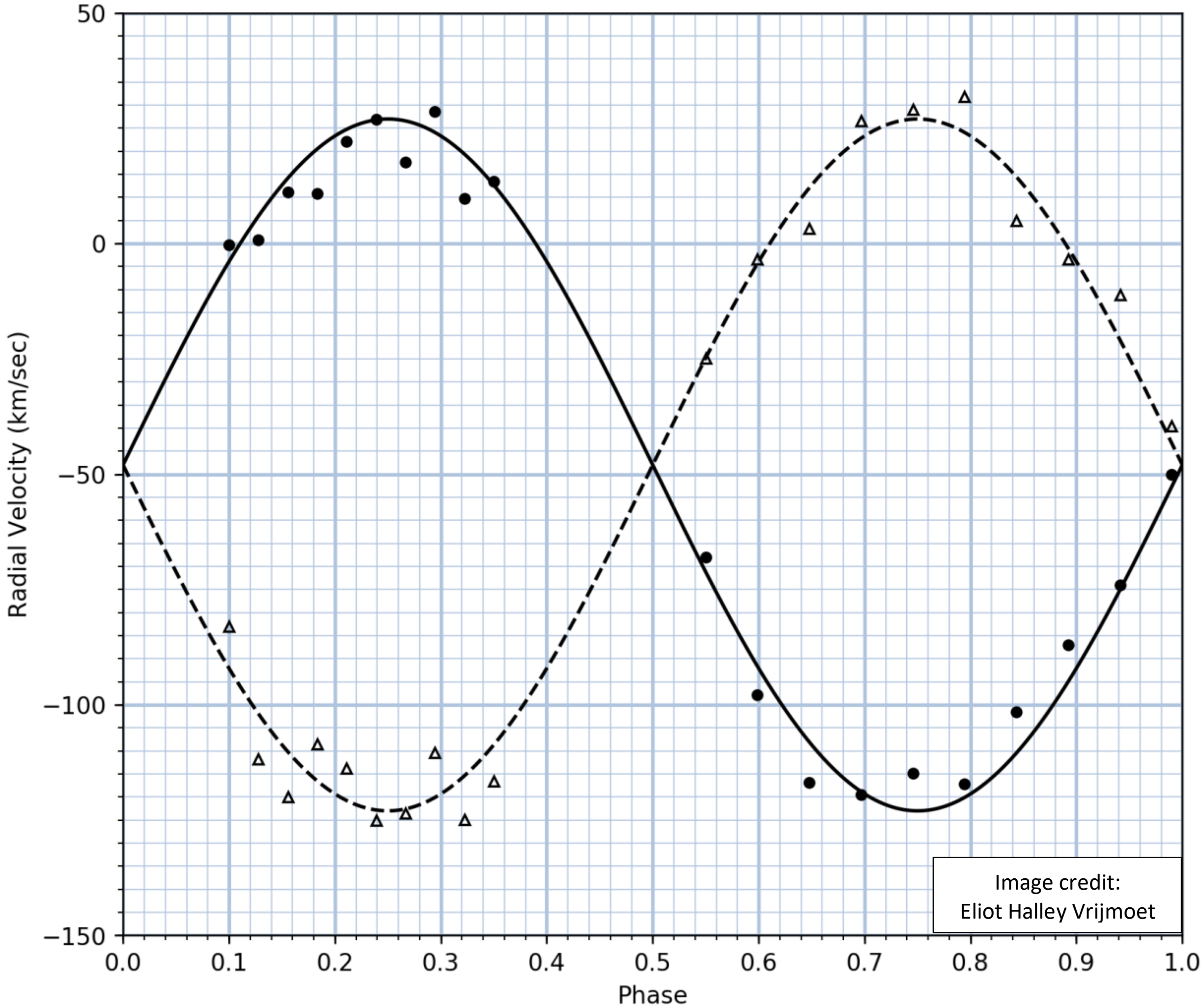


Figure 4: Radial velocity curve for SS Boo. Filled circles represent cooler star and open circles represent hotter star

Repeat the above procedure, except this time measure down to the bottom of the curves and record as β_c and β_h

11. $\boldsymbol{\beta_c}$ = ____________ km/s

12. $\boldsymbol{\beta_c}$ = ____________ km/s

These measurements give us a method of determining the mass ratio of the two stars, but not their actual masses. The mass ratio is given by Equation 4:

$$Mass\ Ratio = \frac{M_h}{M_c} = \frac{\alpha_c + \beta_c}{\alpha_h + \beta_h} \qquad (Equation\ 4)$$

where M_h and M_c are the masses of the hotter and colder star, respectively, and the α and β values are your answers to questions 9-12. Record your mass ratio below:

13. **Mass ratio** = [ ]

Part 3. Dimensions of the SS Bootis System

By combining both the spectroscopic velocities with the photometric properties, the actual dimensions of an eclipsing binary such as SS Boo can be calculated. Thus, it will be possible to calculate the absolute masses, in solar mass units, for each star in the system, and to calculate the absolute radius of each star in solar radius units.

Part A: Stellar Masses

Since SS Boo is a totally eclipsing binary, it can be assumed that the orbital inclination is 90° (edge-on). Thus, the separation between the two stars can be calculated with Equation 5:

$$a = \frac{(0.211V)(P/365.25)}{2\pi} \qquad (Equation\ 5)$$

where *a* is the separation of the stars, *V* is the maximum relative velocity of the two stars, and *P* is the orbital period in days (same as before, *P* = 7.60614 days). The factor of 0.211 converts the velocity from units of km/s to units of AUs/year. It should be obvious that the factor 365.25 converts the orbital period from units of days into years. Therefore, *a* will be in AUs. Use Equation 6 to combine previously determined values of α_h (Question 10) and β_c to solve for *V*:

$$V = \alpha_h + \beta_c \qquad (Equation\ 6)$$

14. **Total relative velocity = *V* =** [ ] km/s

Use this value of *V* and *P* = 7.60614 days to calculate a using Equation 5. Record your answers for *a* and *V* below.

15. **Separation = *a* =** [ ] AUs

The sum of the stellar masses can now be calculated from the values of *a* and *P* using Kepler's Third Law, shown in Equation 7, which states

$$M_h + M_c = \frac{a^3}{(P/365.25)^2} \qquad (Equation\ 7)$$

Record your answer for the sum of the masses below. Your mass units will be in Solar masses (M⊙).

16. **$M_h + M_c$ =** [ ] M⊙

Use the equations for the sum of the masses and the mass ratio, along with your answers for each, to estimate the individual masses of each star in the SS Boo system. Record your answers below

17. **M_h =** [ ] M⊙

18. **M_c =** [ ] M⊙

<u>Part B</u>: Stellar Radii

We can now calculate the absolute radius for each star in terms of the Sun's radius with Equations 8 and 9:

$$R_c = r_c a(214.95) \qquad (Equation\ 8)$$

$$R_h = r_h a(214.95) \qquad (Equation\ 9)$$

where r_c and r_h are your answers to questions 6 & 7, and *a* is your answer to question 15. The factor of 214.95 is the number of solar radii (R⊙) in one AU and converts the radii obtained from AU to solar units. Calculate the value for R_c and R_h for SS Boo, and record your answers below:

19. **R_c =** [ ] R⊙

20. **R_h =** [ ] R⊙