Black Holes

ASTR 1020

Name:

Overview

In this activity, you will explore the basics of black holes, their properties, and how they affect space and time around them.

Objectives

After completing this activity students will be able to:

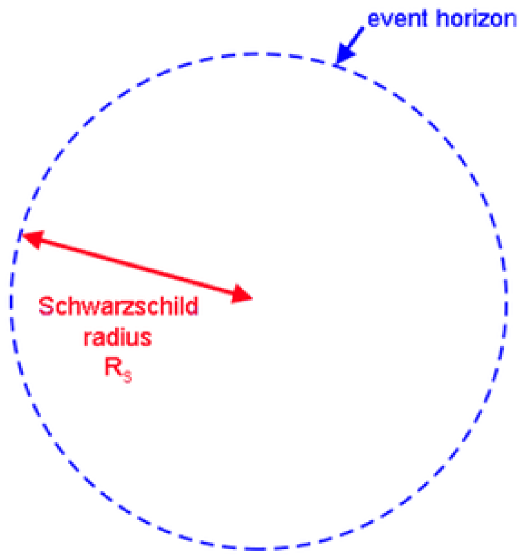
- Calculate the radii, volume, and density of black holes of different masses
- Calculate the effect of time dilation around the gravitational influence of a black hole
- Observe and explain the lensing effect of light around the gravitational field of a black hole
- Observe and explain the effect that lensing and time dilation have on the collapsed matter used to form a black hole

Note: If a question is labeled "THOUGHT QUESTION**" we are looking for you to show critical thinking/justification in your answer, not a "correct" answer**

Definitions

Here are some terms from lecture that we will be using today in lab:

- **Stellar Mass Black Hole** – a black hole with a mass comparable to approximately 5-10 times the Sun's mass.
- **Escape Velocity** – The minimum velocity needed to escape an object's gravitational pull.
- **Event Horizon** – The boundary in spacetime that defines where the escape velocity is equal to the speed of light. Below this boundary, the speed of light is insufficient to escape the surface of the object, therefore nothing can be observed below the boundary.
- **Schwarzschild Radius** – (r_S) The length between the center of a black hole and its event horizon; effectively, the radius of a black hole.
- ***G*** – The gravitational constant, equal to 6.67×10^{-11} m^3 kg^{-1} s^{-2}.
- ***c*** – The speed of light, equal to 3×10^8 m s^{-1}.
- **π** – Equal to 3.14.



- **Supermassive Black Hole** – a black hole with a mass comparable to approximately 10^6 – 10^9 times the Sun's mass.
- **Astronomical Unit** – (AU) The average distance between the Earth and the Sun, equal to 1.5×10^{11} m.
- **Time Dilation** – The difference in elapsed time between two locations, either caused by different velocities or by gravitational fields of differing strengths.
- **Accretion Disk** – A disk of hot matter in a circular orbit around a massive body.

Hint. It might be a good idea to start a spreadsheet file to record values given in the definitions (G, c, π. AU…) and complete your calculations! It is important to show your work in order to get partial credit if something goes wrong with your calculations…

Part 1. Stellar Mass Black Holes

These types of black holes are commonly covered in astronomy lectures. These are the collapsed cores of massive stars which end their life in supernova explosions. The stellar core can no longer use nuclear fusion to hold up the immense gravity and collapses until its escape velocity rises higher than the speed of light. Voila! A black hole is formed.

Part A: The Schwarzschild Radius

The Schwarzschild Radius is defined as:

$$r_s = \frac{2GM}{c^2} \qquad (Equation\ 1)$$

where r_s is the Schwarzschild radius, G is the gravitational constant, M is the mass of the black hole, and c is the speed of light.

1. Let's say we have a black hole with a mass 10 times that of the Sun (the Sun's mass is 2×10^{30} kg, so the mass of the black hole is then 2×10^{31} kg). Using the definitions for G and c, what would the Schwarzschild radius of this black hole be?

2. If the radius of the Sun is 7×10^8 m, how does the black hole's radius compare? (Divide the Schwarzschild radius by 7×10^8 m). Your answer should be in the form of "The black hole is _____ times smaller/bigger than the Sun.

3. If the radius of the Earth is 6×10^6 m, how do the radii compare? (Divide the Schwarzschild radius by 6×10^6 m). Your answer should be in the form of "The black hole is _____ times smaller/bigger than the Earth.

Part B: A spoonful of a black hole

You might often hear scientists compare "a spoonful of…" something and how much it would weigh on Earth. So why don't we do that now? To see how much a spoonful of our black hole would weigh on earth, we need to know its density and its volume. The volume of the black hole is calculated by:

$$V = \frac{4\pi r^3}{3} \qquad (Equation\ 2)$$

Where *r* is your Schwarzschild radius (r_s).

4. Use your Schwarzschild radius from Question 1 to calculate the volume of the black hole.

Now that you know the volume of your black hole, you can now calculate the density as:

$$\rho = \frac{M}{V} \qquad (Equation\ 3)$$

where ρ is the density, *M* is the mass of the black hole and *V* is the volume.

5. Calculate the density of your black hole by plugging in the mass (2×10^{31} kg) and your volume from Question 4.

6. Now that we have our density, we can calculate the mass in a teaspoon. The volume of 1 teaspoon is approximately 5×10^{-6} m^3. Use your density from Question 5 and the volume of a teaspoon to calculate the amount of mass the black hole would have in that small a volume (Hint. Rearrange Equation 3 → $M = \rho V$).

You have calculated the amount of mass that would be in a teaspoon, but now let's convert this to something easily comparable. You're going to calculate how many tons of a black hole are in that one tiny teaspoon.

7. Divide your answer to Question 6 by 1000 to convert kg (which is what you calculated) to tons. Your answer should be in the form of "One teaspoon of this black hole would weigh ___________ tons on Earth."

Part 2. Supermassive Black Holes

Supermassive black holes are exactly what they sound like, really massive. They live in the centers of almost every large galaxy. We'll be looking at two supermassive black holes here, specifically how large they can be depending on their mass.

Part A: Sagittarius A*

Sagittarius A* (pronounced "Sagittarius A star") is the supermassive black hole at the center of our Milky Way. By monitoring the orbits of stars near Sagittarius A* in the Milky Way's bulge for almost 20 years, the mass has been measured to be approximately 4×10^6 times the Sun's mass.

8. a) Use Equation 1 to calculate Sagittarius A*'s Schwarzschild radius in meters **(remember to multiply the Sun's mass by 4×10^6 first!!!).**

   b) Then, divide this number by 1 AU (1.5×10^{11} m) to see how large our supermassive black hole is in terms of the size of the Earth's orbit.

c) Using the Solar System diagram below, if this black hole was placed at the center, how far would it reach with respect to the other planets?

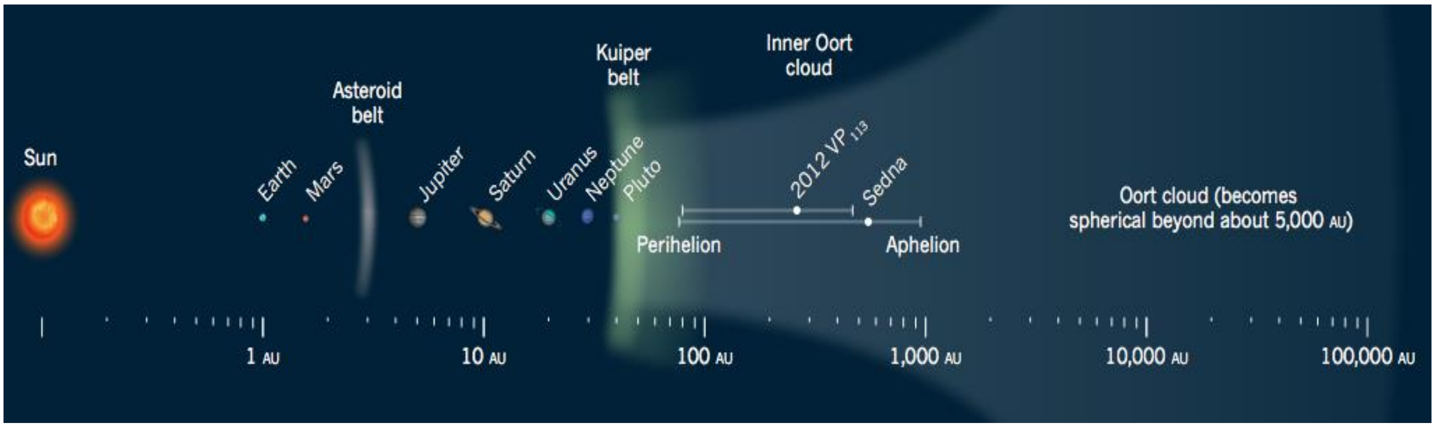


Image credit: www.solarsystemdawing.blogspot.com

Part B: Messier 87

You might remember the first *real* image of a supermassive black hole that has ever been achieved, released by the Event Horizon Telescope Collaboration in 2017 (see image to the right). Its mass was measured to be approximately 6×10^9 times the Sun's mass (1000 times more massive than Sagittarius A*!)

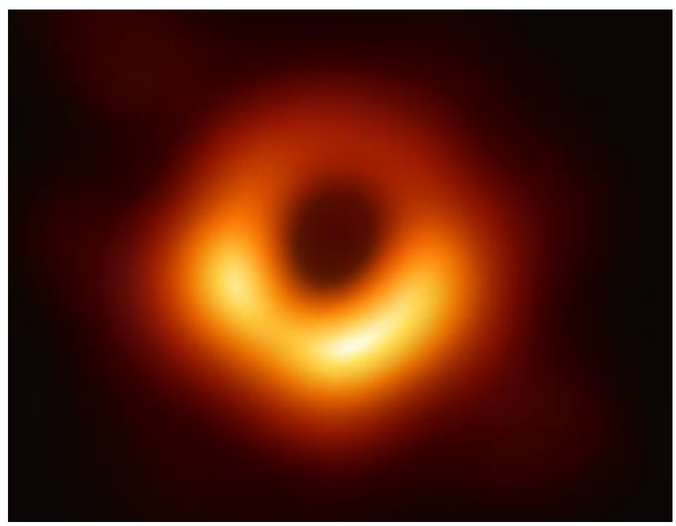


Image credit: Event Horizon Telescope Collaboration

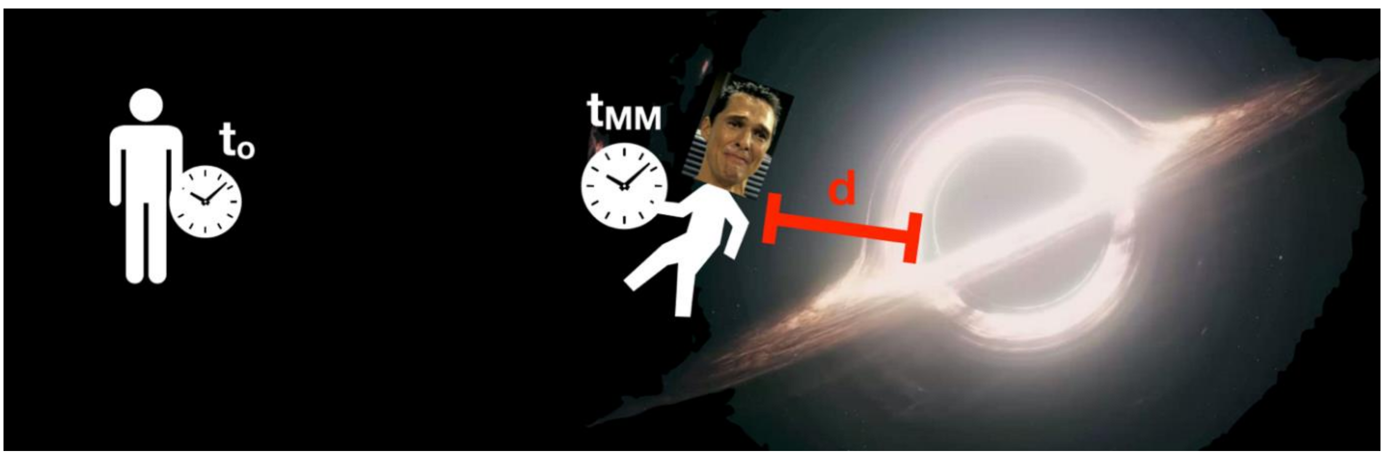
9. a) Do the same calculation as Question 8a, except now multiply the Sun's mass by 6×10^9 before plugging it in to the Schwarzschild radius equation (Equation 1).

b) Now divide this number by 1 AU in meters. How does the radius of this supermassive black hole compare to the Milky Way's?

c) Again, using the Solar System diagram above, if this black hole were placed at the center, how many planets would it swallow up?

Part 3. "Murph!"

A large plot point in *Interstellar* (2014; no spoilers) is the relativity of the passage of time between people on Earth and the astronauts on planets close to a black hole. This is a real phenomenon predicted by the Theory of Relativity. Objects closer to a massive body will experience the flow of time slower than an object further away from the gravitational field. We've measured this very precisely with astronauts aboard the International Space Station. Using high precision atomic clocks, the clock on the ISS moved just slightly faster than a clock on the surface of the Earth.



The equation for time dilation caused by gravity is defined in Equation 4:

$$t_{MM} = t_o\sqrt{1 - \frac{r_s}{d}} \qquad (Equation\ 4)$$

Where t_{MM} is Matthew McConoughey's account of how much time is passing (since he's going to be falling towards the black hole), t_o is your account of how much time is passing (since you are at a safe distance away), r_s is again the Schwarzschild radius, and *d* is Matthew McConoughey's distance away from the black hole.

10. Let's say McConoughey was falling towards Messier 87's supermassive black hole, and that he's currently 100 times further than the Schwarzschild radius. If you both start your clocks at the same time, and yours reads 10 seconds, how much time has passed for McConoughey? (Hint: you don't have to actually plug in Messier 87's Schwarzschild radius. The numerator and denominator will both have a r_s term, so the fraction just simplifies to 1/100).

11. Do the same calculation as Question 10, except now McConoughey is at 1.001 times further than the Schwarzschild radius (again, the fraction simplifies to 1/1.001). How much time has passed for McConoughey now? What is happening to the amount of time passing for McConoughey as he gets closer to the black hole?

Part 4. Trippy Spacetime

Part A: Lensing

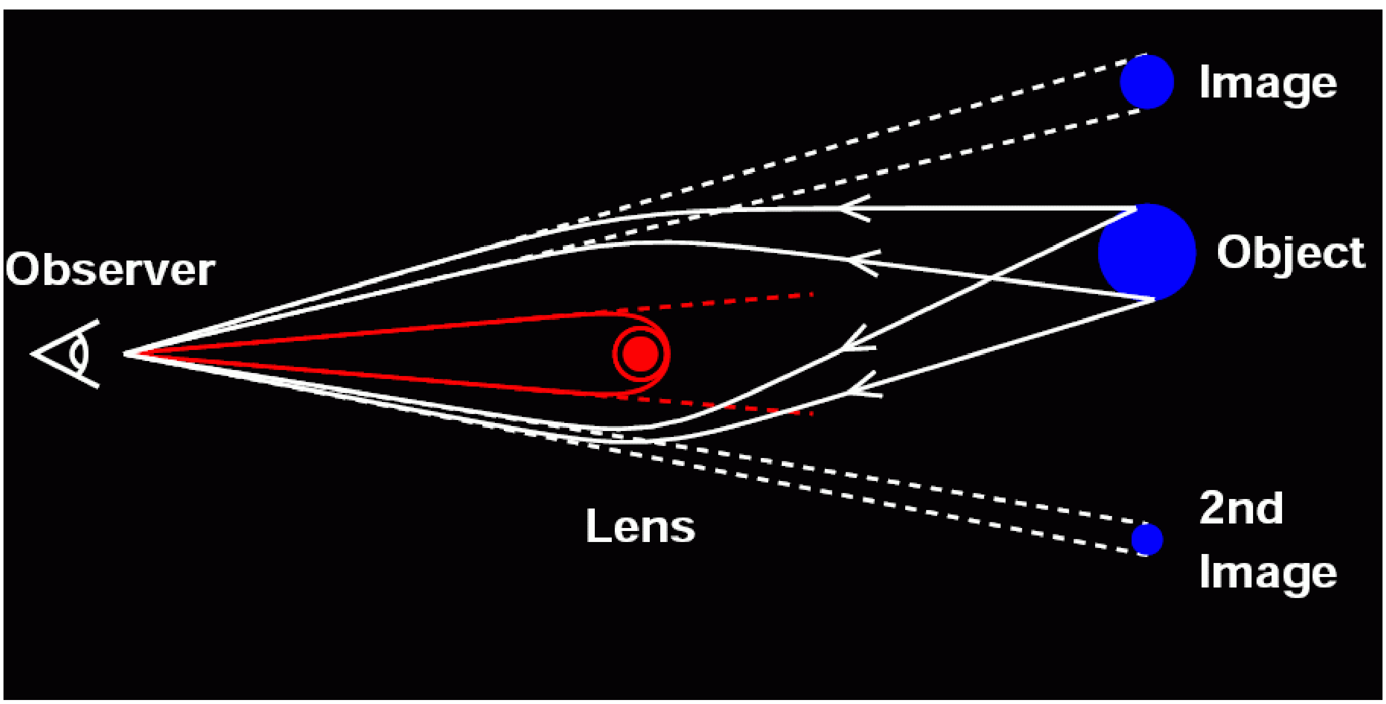


Image credit: https://jila.colorado.edu/~ajsh/insidebh/schw.html

The immense gravity of a black hole doesn't only affect the passage of time, it also warps the fabric of space around it. The diagram below shows light rays emitted from an object that are bent by the massive object in the center. From the observer's point of view, these bent light rays are traced back and make it appear as though there are two more objects (top and bottom right of the diagram). These are not real; however, they are features of the "gravitational lensing" effect that the massive object has on the light rays from the original object.

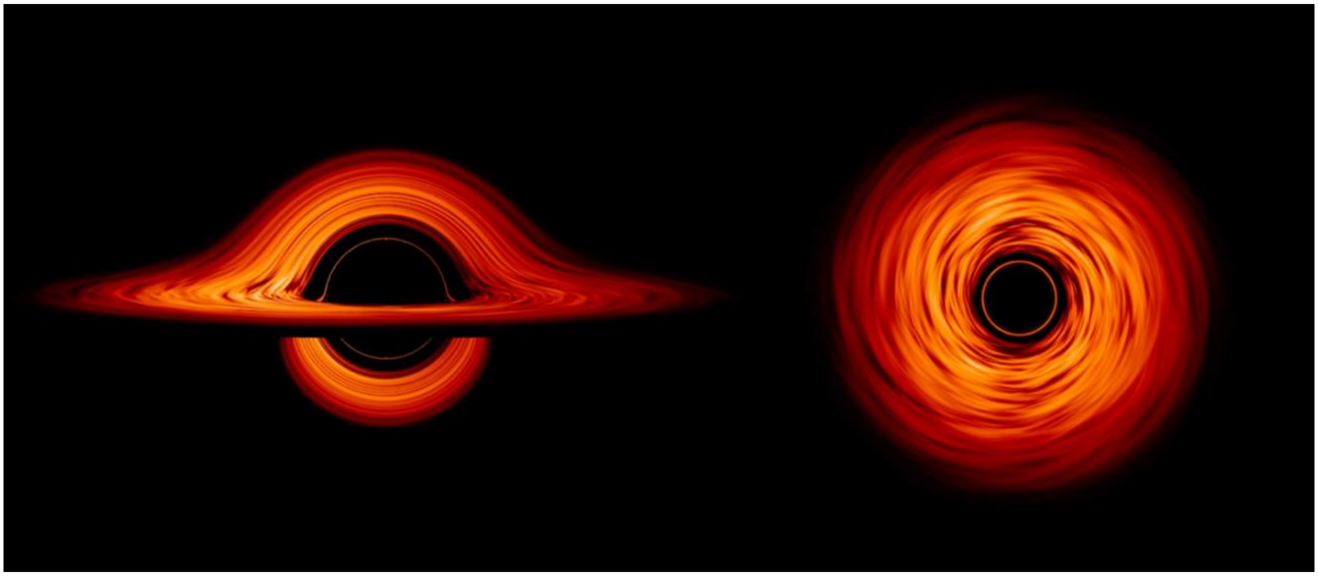


Image credit: Event Horizon Telescope Collaboration

Visit https://svs.gsfc.nasa.gov/13326 and watch the 2nd video on the page to view the lensing effect of a black hole on its accretion disk in different orientations. Below are snapshots of what you are seeing, the **left-hand side is an 'Edge-on' view, the right-hand side is a 'Birds-eye' view.**

12. **THOUGHT QUESTION:** In your own words, using the lensing diagram above, briefly describe what is happening to the image of the accretion disk.

Part B: Dip below the horizon!

Now let's put everything together. You now know how the passage of time is affected by a person getting closer to a black hole, and you know about the warping affect the black hole's gravity causes on the spacetime around it. So, it's finally time to fling yourself through the event horizon!

Visit https://jila.colorado.edu/~ajsh/insidebh/schw.html, and scroll down until you find the tab labeled "Unstable orbit." Click on the gif buttons to view what it would look like to orbit a black hole at 2 times the Schwarzschild radius, and 1.5 times the Schwarzschild radius (gif labeled "Photon sphere"). The event horizon is marked with a red grid, an elapsed time marker is in the bottom right, and your orbit relative to the event horizon is in the bottom left.

Now that your familiar with the trippy imagery of getting closer and closer, click on the gif in the tab labeled "Through the horizon." As you fall through, you start to see something else appear along with the event horizon, and that something is lensed and is always ahead of you as you fall through.

13. **THOUGHT QUESTION:** In your own words, describe what you think is happening. (Hint: remember that a black hole forms from the collapse of massive objects. Collapsing takes time, and given what you know about the passage of time closer to the event horizon, what do you think you are actually seeing?)