Diffuse Interstellar Medium

- Basics, velocity widths
- H I 21-cm radiation (emission)
- Interstellar absorption lines
- Radiative transfer
- Resolved Lines, column densities
- Unresolved lines, curve of growth
- Abundances, depletions
Basics

- Electromagnetic radiation and ISM gas are **not** in local thermodynamic equilibrium (LTE).
- Thus, the populations of atomic and molecular energy levels are **not** specified by LTE.
- A good assumption for low density \( n_H < 10^7 \text{ cm}^{-3} \) gas is that the electrons remain in their lowest energy levels.
- However, collisions between electrons, atoms, and molecules will establish a Maxwellian velocity distribution.

\[
P(v_r) = \frac{1}{\sqrt{\pi b}} e^{-\left(\frac{v_r}{b}\right)^2}
\]

where \( b = \sqrt{\frac{2kT}{m}} \)

- \( b \) = velocity spread parameter, \( T \) = temperature
- \( v_r \) = velocity in one dimension, \( m \) = mass of particle
• Note that the previous equation describes a **Gaussian** profile, normally defined as:

\[
P(v_r) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(v_r/\sigma)^2}
\]

where \(v_r\) = radial velocity, \(\sigma\) = velocity dispersion

• Thus: \(b = \sqrt{2}\sigma\)

• Note the full-width at half-maximum for a Gaussian is:

\[
\text{FWHM} = 2.355 \sigma
\]
Ex) H I 21-cm emission line (1420 MHz)

- What is the FWHM for H I from a cloud of gas at \( T = 50^\circ \text{K} \)?
  
  FWHM \( \approx 1 \text{ km/sec} \)

But what is observed?

- emission profiles are not Gaussian, much broader than thermal width
  - this indicates turbulence
- there are multiple components
  - multiple clouds in the line of sight

Note: \( T_b = \text{brightness temperature} \)

\[
T_b = \frac{c^2}{2k\nu^2} I_\nu \quad \text{(in the Raleigh-Jeans limit)}
\]
What is the emission process for H I 21-cm?

- Radiative transitions between hyperfine levels of the electronic ground state (n=1)
- Upper state: electron and proton spins are parallel, $g_k = 3$ ($g_k =$ statistical weight $= 2S+1$, $S =$ total spin quantum #)
- Lower state: electron and proton antiparallel, $g_j = 1$
- $A_{jk} =$ transition probability $= 2.9 \times 10^{-15}$ sec$^{-1}$
  $\rightarrow$ Lifetime of upper level $= 11$ million years!
- Thus for $n_H \approx 1$ cm$^{-3}$, collisions dominate - levels are populated according to the Boltzmann equation:

$$\frac{n_k}{n_j} = \frac{g_k}{g_j} e^{-\frac{(E_k - E_j)}{kT}} \approx \frac{g_k}{g_j} \approx 3$$

Since the energy difference between levels is very small

- The populations of the levels are essentially independent of temperature in the ISM.
Interstellar Absorption Lines: Radiative Transfer

\[ dF_v = -\kappa_v F_v \, ds + j_v \, ds \]

where \( \kappa_v = \text{opacity}, \quad j_v = \text{emissivity} \)

For UV and optical absorption lines: \( j_v = 0 \)

So: \( dF_v = -\kappa_v F_v \, ds \)

Let: \( d\tau_v = -\kappa_v \, ds \quad \tau_v = \text{optical depth (\# of mean free paths)} \)

\[
\tau_v = \int_0^{\tau_v} d\tau'_v = \int_{F_v}^{F_c} \frac{dF'_v}{F_v} = \ln \left( \frac{F_c}{F_v} \right) \quad \frac{F_v}{F_c} = \exp(-\tau_v)
\]

\( (F_c = \text{continuum flux}, \quad F_v = \text{observed flux}) \)
Can do the same for $\lambda$: $\tau_\lambda = \ln(F_c/F_\lambda)$

Ex) Assume a Gaussian profile in optical depth.

*What is* $(F_\lambda/F_c)$ for $\tau(\lambda_0) = 1, 2, 3, 5$?

Note: These lines are resolved:
FWHM (line) > FWHM (LSF)

LSF – line-spread function (profile of line that is intrinsically infinitely narrow)

**How do we get column densities from absorption lines?**
I. Resolved Lines : FWHM(Line) > FWHM(LSF)

Consider absorption from levels j to k:

$$\kappa_v = n_j s_v \text{ where } n_j = \# \text{ atoms} / \text{cm}^3 \text{ in state } j$$

$$s_v = \text{cross section per frequency}$$

$$\kappa_v = n_j s\Phi_v \text{ where } s = \text{integrated cross section}$$

$$\Phi_v = \text{line profile } (\int \Phi_v dv = 1)$$

$$\tau_v = \int d\tau_v = \int \kappa_v ds' = s \Phi_v \int n_j ds'$$

$$\tau_v = s \Phi_v N_j \quad (= sN_v)$$

If we integrate over frequency:

$$\int \tau_v dv = sN_j \int \Phi_v dv = sN_j \quad (N_j = \text{column density})$$

So:

$$N_j = \frac{1}{s} \int \tau_v dv \quad \text{where } s = \frac{\pi e^2}{m_e c} f_{jk} \quad \text{(Spitzer, Chpt 3)}$$

$$f_{jk} = \text{oscillator strength from lower level } j \text{ to higher level } k$$
Now as a function of $\lambda$:

Note: $\nu = \frac{c}{\lambda}$, $\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$

$N_\lambda d\lambda = N_\nu d\nu$

$N_\lambda = N_\nu \frac{d\nu}{d\lambda} = N_\nu \frac{c}{\lambda^2}$

$N_j = \int N_\lambda d\lambda = \frac{m_e c^2}{\pi e^2} \frac{1}{f_{jk} \lambda^2_{jk}} \int \tau_\lambda d\lambda$

$N_j = 1.1298 \times 10^{20} \frac{1}{f_{jk} \lambda^2_{jk}} \int \tau_\lambda d\lambda \quad (\lambda - \text{Å}, \text{ N - cm}^{-2})$

Thus, for a resolved line [FWHM (line) $>$ FWHM (LSF)]: Determine $\tau_\lambda = \ln \left( \frac{F_c}{F_\lambda} \right)$ and integrate over $\lambda$ to get $N_j$.
• Note: for resolved line, don’t need $W_\lambda$ (EW), assumption of Gaussian distribution, or curve of growth!

II. Unresolved Lines: FWHM(Line) < FWHM(LSF)

\[ W_\lambda = \int (1 - F_\lambda / F_c) \, d\lambda = \int (1 - e^{-\tau_\lambda}) \, d\lambda = \frac{\lambda^2}{c} \int (1 - e^{-\tau_\nu}) \, d\nu \]

1) For unsaturated lines (small \( \tau_\nu \)):

\[ W_\lambda = \frac{\lambda^2}{c} \int \tau_\nu \, dv = \frac{\lambda^2}{c} \frac{\pi e^2}{m_e c} f_{jk} N_j \quad (\lambda - \text{Å}, \ W_\lambda - \text{Å}, \ N - \text{cm}^{-2}) \]

Thus: \[ N_j = 1.1298 \times 10^{20} \frac{1}{f_{jk} \lambda_{jk}^2} W_\lambda \]

\[ \frac{W_\lambda}{\lambda_{jk}} = \frac{\pi e^2}{m_e c} N_j \lambda_{jk} f_{jk} = 8.85 \times 10^{-13} N_j \lambda_{jk} f_{jk} \]

- This is the linear part of the curve of growth.
2) What is $W_\lambda$ for unresolved, saturated lines? ($\tau > 1$)

- Assume a Maxwellian velocity distribution and Doppler broadening
- The redistribution of absorbed photons in frequency is:

$$\Phi_v = \lambda_{jk} P(v_r) = \frac{\lambda_{jk}}{\sqrt{\pi b}} e^{-(v_r/b)^2}$$

$$\frac{W_\lambda}{\lambda_{jk}} = \frac{\lambda_{jk}}{c} \int (1 - e^{-\tau_v}) \, dv \quad \text{where:} \quad \tau_v = s \Phi_v N_j$$

It can be shown that:

$$\frac{W_\lambda}{\lambda_{jk}} = \frac{2bF(\tau_0)}{c}, \quad \text{where} \quad F(\tau_0) = \int_0^\infty [1 - \exp(-\tau_0 e^{-x^2})] \, dx$$

where:

$$\tau_0 = \frac{N_j s\lambda_{jk}}{\sqrt{\pi b}} = \frac{1.497 \times 10^{-2}}{b} N_j \lambda_{jk} f_{jk}$$

($\tau_0$ is optical depth at line center, parameters in cgs units)
- So $W_\lambda = \text{fct} (N,b)$ for a given line $(\lambda, f)$
- $F(\tau_0)$ is tabulated in Spitzer, Ch. 3, page 53
- For large $\tau_0$: $F(\tau_0) = (\ln \tau_0)^{1/2}$
- This is the flat part of the curve of growth.

3) For very large $\tau_0$, damping wings are important:

$$\frac{W_\lambda}{\lambda_{jk}} = \frac{2}{c} (\lambda_{jk}^2 N S \delta_k)^{1/2}$$

where $\delta_k = \text{radiation damping constant}$

- This is the square root part of the COG, which is only important for very high columns (e.g., Ly$\alpha$ in the ISM).
- The most general COG $(2 + 3)$ uses a Voigt intrinsic profile (Gaussian + Lorentzian)
To generate curves of growth (Case 2):

- For a given $b$ and $N\lambda f$, determine $\tau_0, F(\tau_0)$, and then $W_\lambda/\lambda$
- Do this for different $b$ values (km/sec) to get a family of curves:
Ex) O VII Absorption in Chandra Spectrum of NGC 5548
(Crenshaw, Kraemer, & George, 2003, ARAA, 41, 117)

- FWHM (LSF) $\approx 300$ km/sec, observed FWHM only slightly larger
- Plot the standard curve of growth (COG) for different $b$ values
- Assume $N$(O VII) and overplot $\log(\text{EW}/\lambda)$ vs. $\log(Nf\lambda)$
- Try different $N$ (O VII) until you get a match to a particular $b$. 
Curves of Growth

$N(\text{O VII})$

$b = 200 \ (\pm 50) \ \text{km/sec}, \ N(\text{O VII}) = 4 \ (\pm 2) \times 10^{17} \ \text{cm}^{-2}$
Ex) Depletion in ISM clouds (see Spitzer, page 55)

- Lines from ions expected to appear in the same clouds are shifted horizontally until a “b” value is obtained $\rightarrow N(\text{ion})$
Application: Abundances

Cosmic Abundance of element x: \( A(x) = 12.0 + \log \left( \frac{N_x}{N_H} \right)_{\text{cosmic}} \)

Depletion of element x: \( D(x) = \log \left( \frac{N_x}{N_H} \right)_{\text{cloud}} - \log \left( \frac{N_x}{N_H} \right)_{\text{cosmic}} \)

(Note: cosmic abundances usually means solar abundances)

Cosmic Abundances and Depletions Toward \( \zeta \) Oph
(from Spitzer, page 4)

| Element | He  | Li  | C  | N  | O  | Ne | Na | Mg | Al | Si | P  | S  | Ca | Fe  |
|---------|-----|-----|----|----|----|----|----|----|----|----|----|----|-----|
| A(x)    | 11.0| 3.2 | 8.6| 8.0| 8.8| 7.6| 6.3| 7.5| 6.4| 7.5| 5.4| 7.2| 6.4 | 7.4 |
| D(X)    | -1.5| -0.7| -0.7| -0.6| -0.9| -1.5| -3.3| -1.6| -1.1| -0.3| -3.7| -2.0|
- Depletions indicate condensation of elements out of gas phase onto dust grains
- The most refractory elements (highest condensation temperatures) are the most depleted (due to formation in cool star atmospheres)
More Recent Depletions
(ζ Oph – Dopita, p. 65)
Gas-Phase Depletions
(Savage & Sembach, 1996, ARAA, 34, 279)

- Dust grains in halo clouds are destroyed by shock fronts from supernova remnants
The Multiphase *Diffuse* Interstellar Medium  
(Dopita, Chapter 14)

- Observations by Copernicus and IUE indicate highly-ionized gas (C IV, N V, O VI) in the ISM.
- McKee & Ostriker (1977, ApJ, 218, 148) proposed a five-phase model, which is the currently accepted one.
- Each phase is in rough pressure equilibrium ($n_H T \approx 2000 – 6000 \text{ cm}^{-3} \text{ K}$)

1) The molecular medium (MM)  
2) The cold neutral medium (CNM)  
3) The warm neutral medium (WNM)  
4) The warm ionized medium (WIM) (i.e., H is mostly ionized)  
5) The hot ionized medium (HIM)
<table>
<thead>
<tr>
<th>Phase</th>
<th>$n_H$(cm$^{-3}$)</th>
<th>T (°K)</th>
<th>h (kpc)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM</td>
<td>$\geq 10^3$</td>
<td>20</td>
<td>0.05</td>
<td>CO, HCN, H$_2$O emission, H$_2$ abs.</td>
</tr>
<tr>
<td>CNM</td>
<td>20</td>
<td>100</td>
<td>0.1</td>
<td>H I 21-cm emission, H$_2$, C II, Si II, Mg II, etc. absorp.</td>
</tr>
<tr>
<td>WNM</td>
<td>1.0</td>
<td>6000</td>
<td>0.4</td>
<td>H I 21-cm emission, C II, Si II, Mg II absorp. (no H$_2$)</td>
</tr>
<tr>
<td>WIM</td>
<td>0.3</td>
<td>10,000</td>
<td>1</td>
<td>H$\alpha$ emission, Al III, Si IV, C IV absorp.</td>
</tr>
<tr>
<td>HIM</td>
<td>$10^{-3}$</td>
<td>$10^6$</td>
<td>10</td>
<td>Soft X-ray emission, C IV, N V, O VI absorp.</td>
</tr>
</tbody>
</table>

- Scale height given by: $n_H = n_0 e^{-z/h}$, $z =$ height above Galactic plane (Savage, 1995, ASP Conf. Series, 80, 233)
- Ionization increases with increasing $z$
- Depletion decreases with increasing $z$
- Hot phase driven by supernova remnants (shocks destroy dust grains)
What are these phases?

1) MM: self-gravitating molecular clouds <1% of the volume (but ~50% of ISM mass)
2) CNM: only 5% of the volume, sheets or filaments in the ISM
3) WNM: Photodissociation regions (PDRs), hot dust
4) WIM: ~25% of the volume together with WNM, ionized by O stars, SNRs (shocks and cosmic rays)
5) HIM: ~70% of the volume, driven into halo by SNRs
   - heated by shocks, cosmic rays
   - coalesce to form superbubbles, fountains, “chimneys”
   - hot ($10^6$ K) gas in Galactic halo (O VI absorption)

• McKee and Ostriker model: MM and CNM are dense clouds that are surrounded by WNM and WIM halos, embedded in the HIM
• O VI absorption in halo can also be from infalling gas from IGM (cosmic web) (Sembach et al. 2003, ApJS, 146, 155).