Photoionized Gas – Ionization Equilibrium

- Ionization
- Recombination
- H nebulae - case A and B
- Strömgren spheres
- H + He nebulae
- Heavy elements, dielectronic recombination
- Ionization structure

Ionization Equilibrium Equation

- Start with a pure H nebula
- # photoionizations/vol/sec = # recombinations/vol/sec:

$$n_{H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\,a_\nu(H^0)\,d\nu = n_e n_p \alpha(H^0,T) \quad (h\nu_0 = 13.6 \text{ eV})$$

$$\text{where: } J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega = \text{mean intensity} \ (\text{ergs s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}\text{sr}^{-1})$$

$$a_\nu = \text{ionization cross section}$$

$$\alpha = \text{recombination coefficient for H } (\text{cm}^3\text{s}^{-1})$$

Ex) For a pure H nebula: $n_p = n_e,\ n_H = n_{H^0} + n_p$

At a distance of 5 pc from an O7 star (T = 40,000 K)

If $n_H = 10 \text{ cm}^{-3} \rightarrow n_{H^0}/n_H \approx 4 \text{ x } 10^{-4}$

- Using the same parameters, when the hydrogen starts to go neutral, the thickness of the H II region is:

$$d \approx \frac{1}{n_{H^0} a_\nu} \approx 0.01 \text{ pc}$$

- So the H II region is almost completely ionized until it reaches a "transition region", which is very thin
- Such an idealized case is called a Strömgren sphere

- Above calculations depends on the atomic parameters: a_ν, α and the details of atomic structure:

Ex) Hydrogen ground state: $\boxed{1^2\mathrm{S}}$ (or generally 1s ^{2}S)

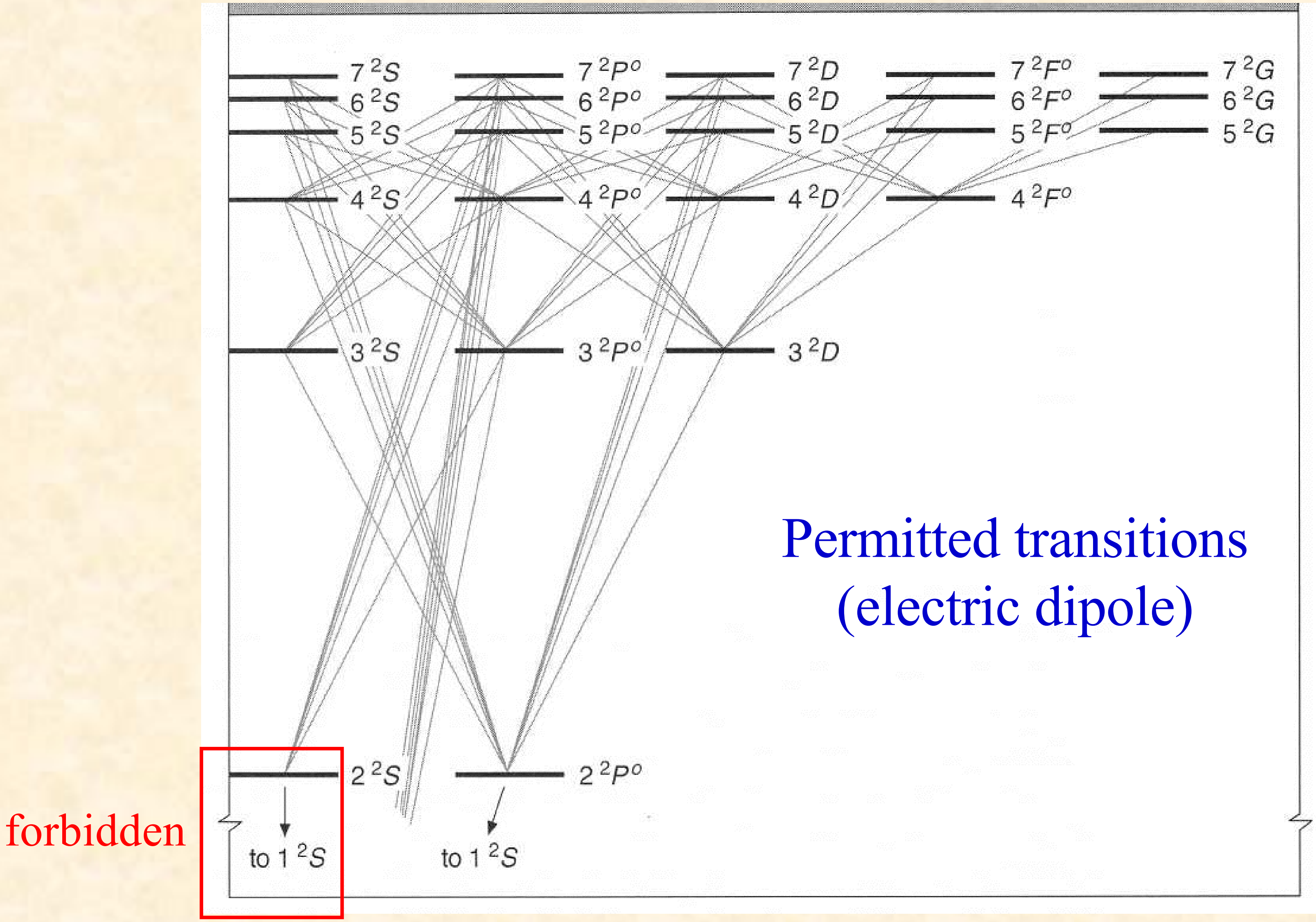
n = 1 (principal quantum number)

L = 0 (total angular momentum number)

L = 0, 1, 2, 3 → S, P, D, F

m = 2 = 2S+1 (where S=spin = ½) (m =multiplicity when L > S)

Energy-Level Diagram for H I



(Osterbrock & Ferland, p. 19)

Atomic Parameters: Transition Probabilities

For permitted transitions of H I ($\Delta L = \pm 1$):

$A_{nL,n'L'} = 10^4 - 10^8 s^{-1}$, so lifetimes of states $\tau \approx 10^{-4} - 10^{-8}$ sec

Ex) $2^2P \rightarrow 1^2S$ gives Lα photon at 1216 Å ($\tau \approx 10^{-8}$ sec)

Using previous example (O7 star at 5 pc):

$$\#\text{ionizations} / H^0 = \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H^0)\, d\nu \approx 10^{-8} s^{-1}$$

So there are 10^8 sec between ionizations of an H atom

$\rightarrow$ all H^0 atoms are essentially in ground state

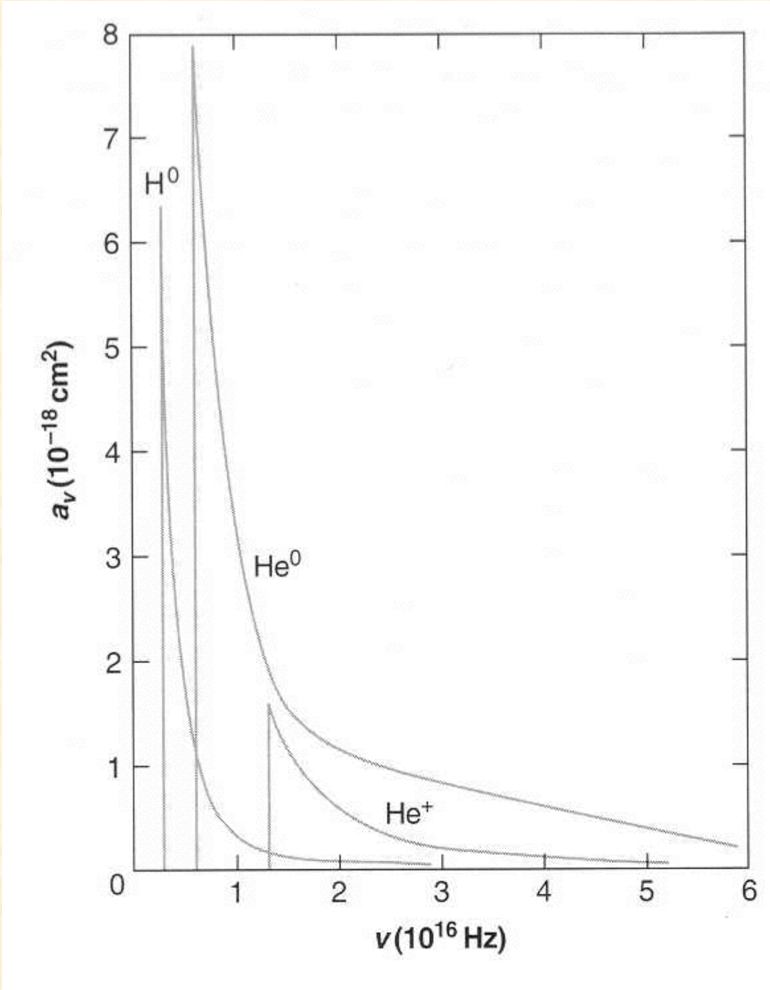
Ex) $2^2S \rightarrow 1^2S$ gives two-photon continuum: $\tau \approx 0.12$ sec

$\rightarrow$ Forbidden transitions are common in nebulae

Photoionization Cross Sections

- For a hydrogenic ion, with charge Z:

$$a_\nu(Z) \sim Z^{-2}\nu^{-3} \text{ (approximately) for } h\nu > h\nu_1 (= 13.6\ Z^2 \text{ eV})$$



(Osterbrock & Ferland, p. 21)

Ionization Potentials:

IP (H^0) = 13.6 eV

IP (He^0) = 24.6 eV

IP (He^+) = 54.4eV

Recombination Coefficients

- Ejected electrons set up a Maxwell-Boltzman distribution:

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

Recombination to level n, L :

$$\alpha_{nL}(H^0,T) = \int_0^\infty v\,\sigma_{nL}(H^0,v)\,f(v)\,dv$$

$\alpha_{nL} \propto T^{-1/2}$ (approximately, Osterbrock p. 21)

- Total recombination coefficient:

$$\alpha_A = \sum_{nL} \alpha_{nL}(H^0,T)$$

At T = 10,000 K: $\alpha_A = 4.18 \text{ x } 10^{-13}\ \text{cm}^3\text{sec}^{-1}$ (Osterbrock, p. 22)

Recombination time: $\tau_{rec} = \dfrac{1}{n_e \alpha_A} \approx \dfrac{10^5}{n_e}$ yrs

Radiation Field

Ex) Consider a pure H nebula around a single star:

$$J_\nu = J_{\nu\ stellar} + J_{\nu\ \ diffuse}$$

$$4\pi J_{\nu\ stellar} = \frac{L_\nu}{4\pi r^2}\ e^{-\tau_\nu}$$

where $\tau_\nu = \int_0^r n_{H^0}\left(r'\right)\ a_\nu\ dr'$ at location r

$\boxed{\text{For a pure H nebula, } 4\pi J_{\nu \text{ diffuse}} = \text{ionizing flux from recombination}}$

Case A) Optically thin nebula: $J_{\nu\ \ \text{diffuse}}\ \approx\ 0$ (radiation escapes)

$$n_{H^0}\int_{\nu_0}^{\infty}\frac{L_\nu}{4\pi r^2 h\nu}\ a_\nu e^{-\tau_\nu}\ d\nu = n_e n_p \alpha_A(H^0,T)$$

where $\alpha_A = \sum_{n=1}^{\infty}\alpha_n(H^0,T)$

Case B) Optically thick nebula: $\rightarrow$ Use "on the spot" approximation: recombinations to ground generate photons that are absorbed locally

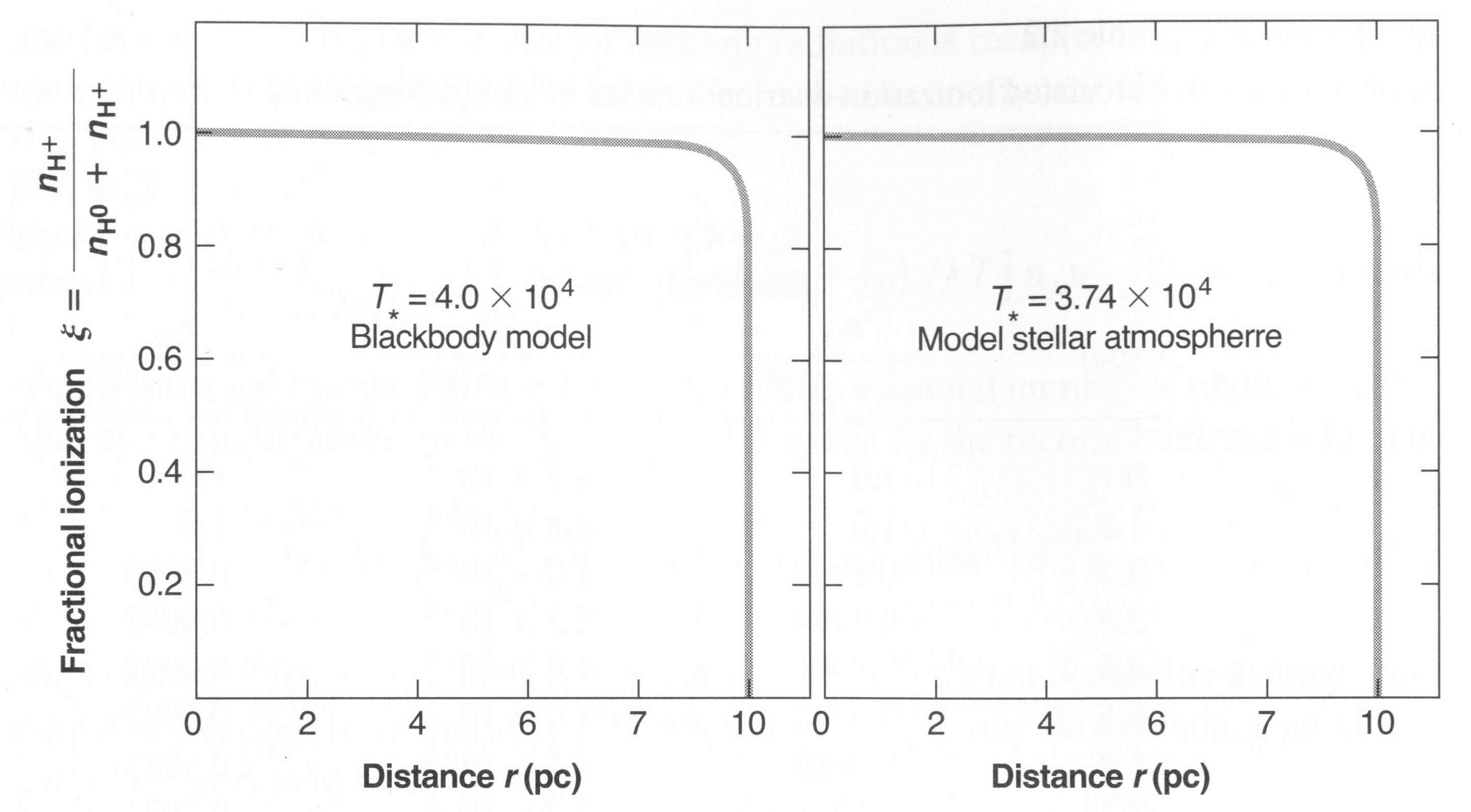
$$n_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_{\nu\ diffuse}}{h\nu}\ a_\nu\ d\nu = n_e n_p \alpha_1(H^0, T)$$

Thus: $$n_{H^0} \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu}\ a_\nu e^{-\tau_\nu}\ d\nu = n_e n_p \alpha_B(H^0, T)$$

$$\alpha_B = \sum_{n=2}^{\infty} \alpha_n(H^0, T)$$

- Nearly all nebulae are Case B: optically thick to ionizing radiation
- Since τ is a function of (n_{H^0}, r), $n_e = n_p$, and $n_H = n_p + n_H{}^0$, the ionization equilibrium equation above can be integrated outward for a given $n_H(r)$ and $T(r)$ to get $n_p/n_H(r)$
- This is a crude model, since $T(r)$ is not known a priori.

Structure of Pure H Nebula:Models



(Osterbrock & Ferland, p. 26)

- assuming $n_H = 10$ cm^{-3} and T = 7500 K, constants as function of radius

Global Ionization Balance

Consider a Stromgren sphere (pure H nebula around a single star):

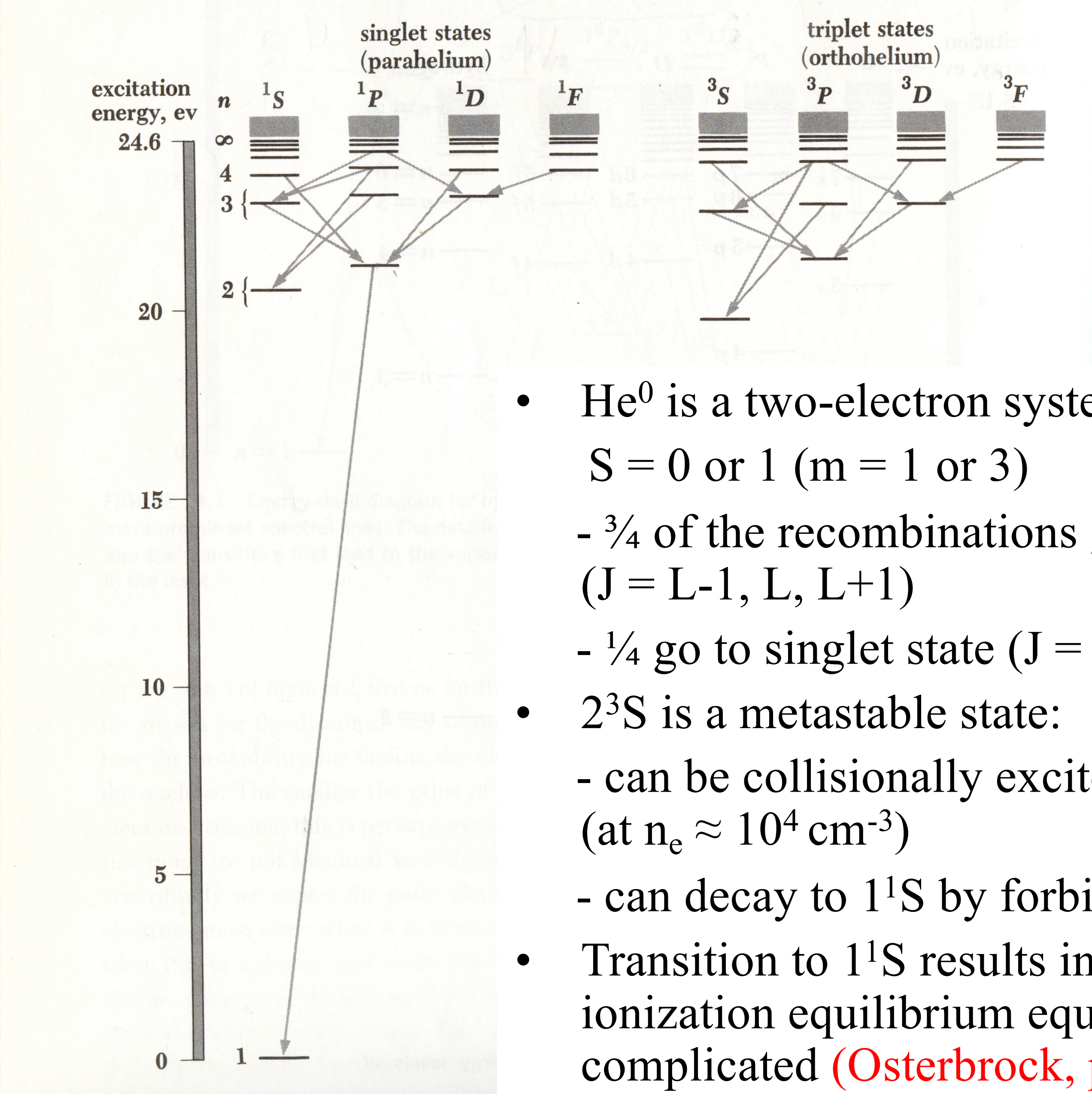
$$Q(H^0) = \#\text{ionizing photons emitted by star per second}$$

$$Q(H^0) = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\, d\nu = \tfrac{4}{3}\pi r_1^3 n_H^2 \alpha_B \;\rightarrow r_1 = \text{radius of Stromgren sphere}$$

Spectral type	T_* (K)	M_V	log $Q(H^0)$ (photons/s)	log $n_e n_p r_1^3$ n in cm^{-3}; r_1 in pc	log $n_e n_p r_1^3$ n in cm^{-3}; r_1 in pc	r_1 (pc) $n_e = n_p = 1$ cm^{-3}
O3 V	51,200	−5.78	49.87	49.18	6.26	122
O4 V	48,700	−5.55	49.70	48.99	6.09	107
O4.5 V	47,400	−5.44	49.61	48.90	6.00	100
O5 V	46,100	−5.33	49.53	48.81	5.92	94
O5.5 V	44,800	−5.22	49.43	48.72	5.82	87
O6 V	43,600	−5.11	49.34	48.61	5.73	81
O6.5 V	42,300	−4.99	49.23	48.49	5.62	75
O7 V	41,000	−4.88	49.12	48.34	5.51	69
O7.5 V	39,700	−4.77	49.00	48.16	5.39	63
O8 V	38,400	−4.66	48.87	47.92	5.26	57
O8.5 V	37,200	−4.55	48.72	47.63	5.11	51
O9 V	35,900	−4.43	48.56	47.25	4.95	45
O9.5 V	34,600	−4.32	48.38	46.77	4.77	39
B0 V	33,300	−4.21	48.16	46.23	4.55	33
B0.5 V	32,000	−4.10	47.90	45.69	4.29	27

(Osterbrock & Ferland, p. 27)

Adding He: Energy Levels for He^0



- He^0 is a two-electron system:

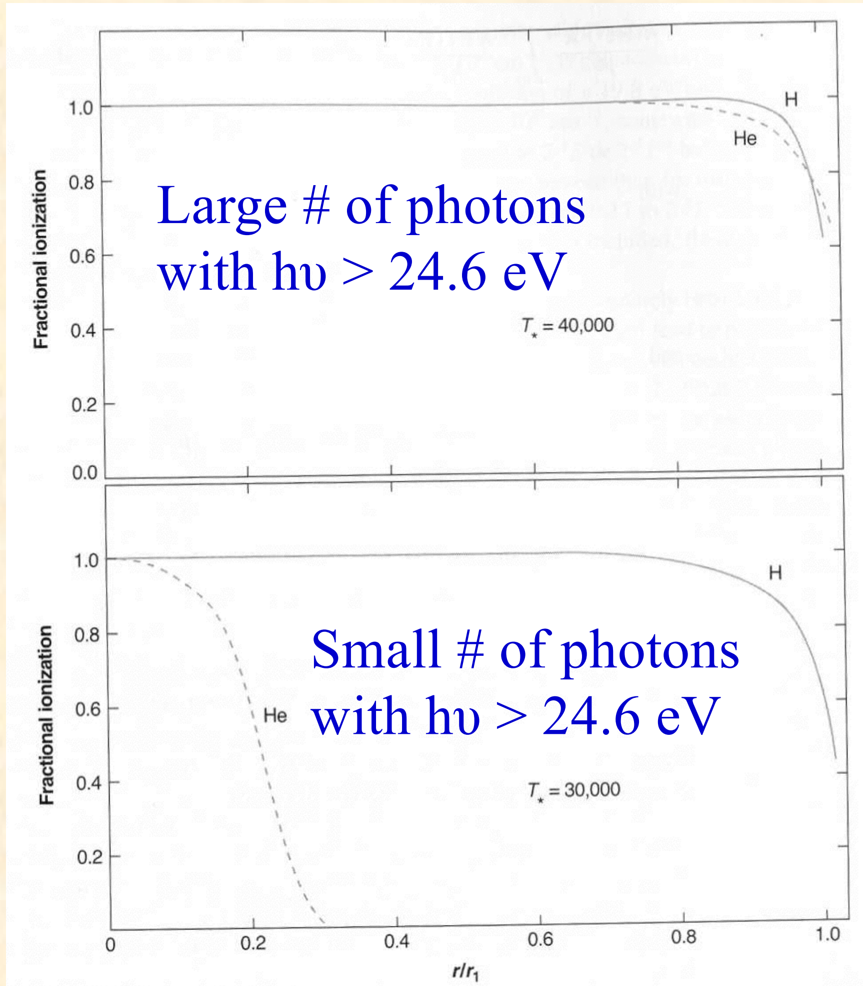
  S = 0 or 1 (m = 1 or 3)

  - ¾ of the recombinations go to triplet state (J = L-1, L, L+1)
  - ¼ go to singlet state (J = L)

- 2^3S is a metastable state:
  - can be collisionally excited to 2^1S or 2^1P (at $n_e \approx 10^4\ cm^{-3}$)
  - can decay to 1^1S by forbidden photon at 19.8 eV

- Transition to 1^1S results in H-ionizing photon: ionization equilibrium equations get more complicated (Osterbrock, p. 30)

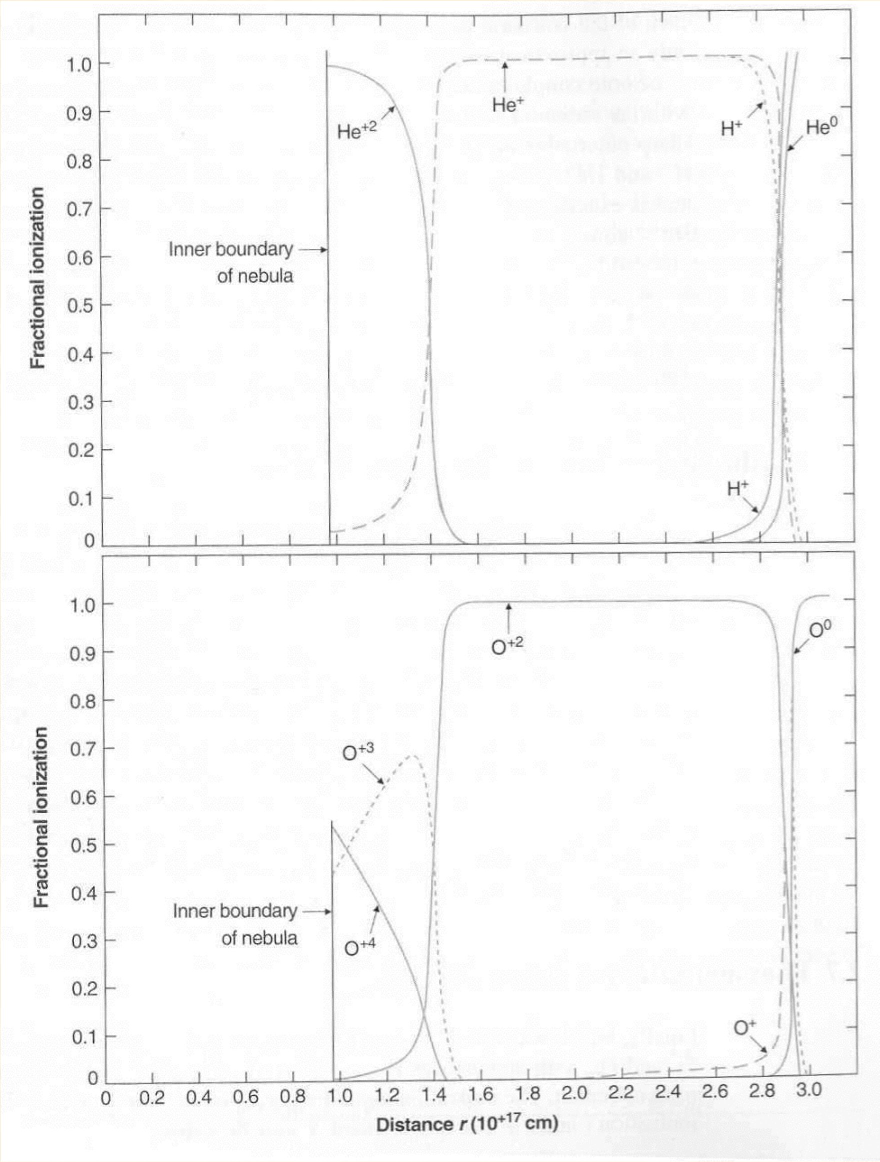
H + He Nebula Models



(Osterbrock & Ferland, p. 32)

- Due to its abundance, H controls the full extent of the ionized zone
- For very hot (O) stars, H^+ and He^+ zones are coincident
- For an H II region, few photons have hν > 54.4 eV → no He^{+2} zone

Planetary Nebula Model
- ionization of He^+ important



(Osterbrock and Ferland, p. 36)

Photoionization of Heavy Elements

- \# ionizations of X^i = # recombinations of X^{i+1}

  $n(X) = n(X^0) + n(X^+) + \ldots + n(X^{+n})$

  where $n(X)/n(H)$ = fractional abundance of element X
- Heavy element contribution to diffuse field ≈ 0
- a_ν, $\alpha(T)$ not well known for many ions
- a_ν can show complicated structure, due to resonances.
- There can also be ionizations of inner shell electrons from X-rays (important in AGN). Vacancies are filled with outer electrons:
  1) Producing X-ray emission lines and/or
  2) Auger effect: one or more electrons move down, one or more are ejected from the atom.

Recombination of Heavy Elements

- Dielectronic recombination:
- Larger than normal recombination for some ions
- Relative importance tends to increase with Temperature
- Captured free electron excites bound electron in atom, leading to a "double excited" state
- Both electrons eventually decay to ground state

- Charge exchange
- Occurs when two ions have about the same I.P.
- Example:

$$O^0 + H^+ \rightarrow O^+ + H^0$$

(works in reverse as well)

$$\frac{n(O^0)}{n\left(O^+\right)} \approx \frac{n(H^0)}{n\left(H^+\right)} \text{ (O ionization locked to H)}$$

“Real” Photoionization Models

The input parameters are typically:

1) Input energy spectrum (type of star, AGN, etc.)
2) Ionizing flux or luminosity plus distance
3) Density of hydrogen (ionized + neutral)
4) Abundances of the elements

- most simple models assume plane-parallel geometry (slab) and constant density in slab.

The output parameters include:

1) Ionization structure into the slab.
2) Temperature structure of the slab.
3) Emission line fluxes or ratios