Thermal Equilibrium

- Energy conservation equation
- Heating by photoionization
- Cooling by recombination
- Cooling by brehmsstralung
- Cooling by collisionally excited lines
- Collisional de-excitation
- Detailed Balancing
- Critical Densities
- Comparison of heating and cooling rates

Energy conservation

- Heating provided by photoionization of electrons
- Cooling provided by:

1) recombination lines (mostly H, He)

2) brehmsstralung (free-free radiation)

3) collisional excitation of heavy ions, and subsequent radiation

- Thermal equilibrium:

$$\boxed{G = L_r + L_{ff} + L_c}$$

(G – energy gained by photoionization, L – energy lost by radiation due to the above processes)

- Ionization: creates an electron with energy $\frac{1}{2} mv_i^2 = h(\nu - \nu_0)$
- Recombination: electron gives up energy = $\frac{1}{2} mv_f^2$
- Net energy that goes into heating: $\frac{1}{2} mv_i^2 - \frac{1}{2} mv_f^2$

Heating

- Heating provided by photoionization of electrons
- For a pure hydrogen nebula:

$$G(H) \;=\; \text{energy input/vol/sec} \quad (\text{ergs s}^{-1}\text{cm}^{-3})$$

$$\boxed{G(H) \;=\; n_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu - \nu_0)\, a_\nu(H^0)\, d\nu}$$

$$= n_e n_p \;\alpha_A(H^0, T)\; \tfrac{3}{2} kT_i \quad (\text{using ionization equil.})$$

T_i is the initial electron temperature

- T_i (electron temperature) is low close to the star, as $h\nu_0$ photons are absorbed in the inner nebula first
- However, as τ increases, T_i increases due to absorption of photons with energies > $h\nu_0$

Energy Loss by Recombination

$$L_R(H) = n_e n_p \, kT \, \beta_A(H^0,T)$$

where β_A = recomb. coeff. averaged over kinetic energy

$$\beta_{nL}(H^0,T) = \frac{1}{kT}\int_0^\infty v \, \sigma_{nL}(H^0,v) \, f(v) \, \tfrac{1}{2} mv^2 dv$$

On the spot approximation:

$$\boxed{L_R(H) = n_e n_p kT \beta_B(H^0,T)}$$

where β_B summed over all states but ground

For a pure H Nebula:

$G(H) \approx L_R(H)$ (free-free radiation not very important)

For recombination of He:

-same formulae as for heating and cooling of H

Recombination of other elements:

-usually not important, since heating and cooling proportional to ionic densities (and abundances are low)

Brehmsstralung (Free-Free Radiation)

$$\boxed{L_{ff} = 1.42 \times 10^{-27} Z^2 T^{1/2} g_{ff} n_e n_+}$$

where g_{ff} = Gaunt factor, weak function of n_e and T

- quantum mechanical correction for classical case
- between 1.0 and 1.5 for H II regions

- brehmsstralung usually not very important at nebular temperatures; recombination and collisional excitation dominates

(but dominant cooling mechanism in $T = 10^7 - 10^8$ K intracluster gas)

Collisionally Excited Radiation

- Dominated by collisional excitation of low-lying levels of heavy elements (e.g., O^+,O^{++},N^+)
- Excited levels are mostly metastable, which result in forbidden or semi-forbidden lines (low A values)
- $\Delta E \approx kT$, so very important coolants, despite lower abundance
- Consider two levels: lower (1) and upper (2)
- Collision cross-section:

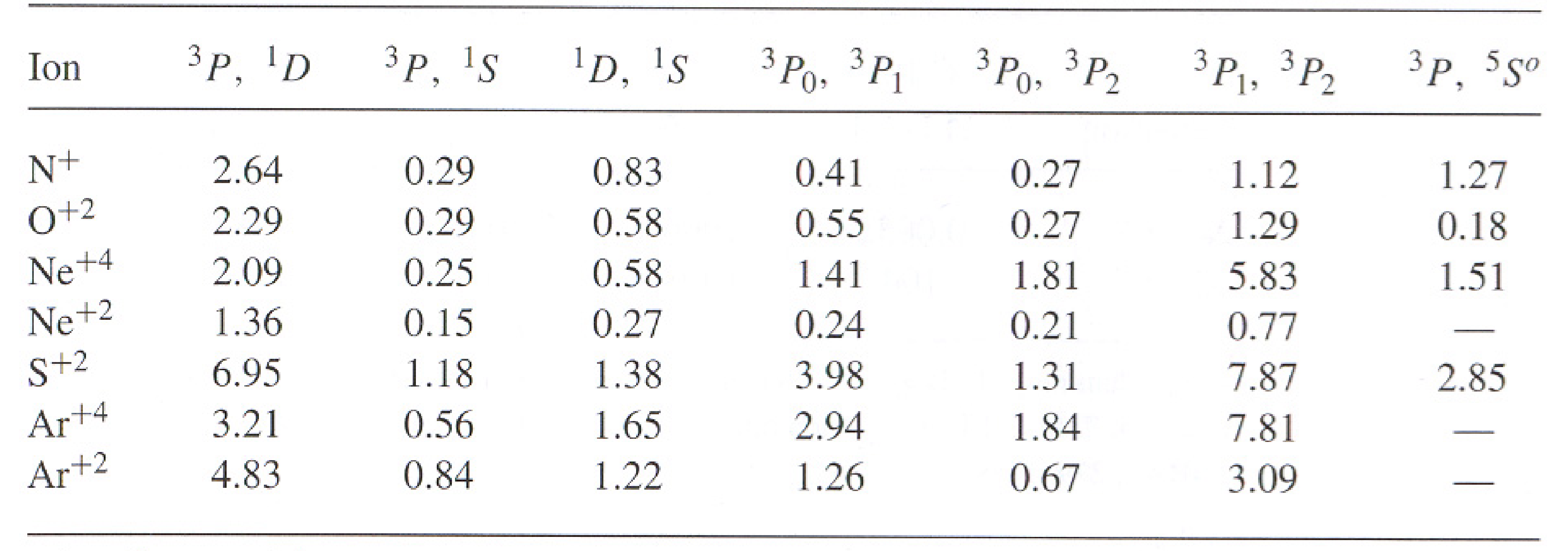
$$\sigma_{12}(v) = \frac{\pi h^2}{m^2 v^2} \frac{\Omega_{12}}{\omega_1} \quad (\text{for } \tfrac{1}{2} m v^2 > \chi \quad \text{where } \chi = h\nu_{12})$$

where Ω_{12} = collision strength from levels 1 to 2

(essentially constant with temperature at these electron velocities)

$\omega_1 =$ statistical weight for level 1

Collision Strengths



Ion	$^3P,\ ^1D$	$^3P,\ ^1S$	$^1D,\ ^1S$	$^3P_0,\ ^3P_1$	$^3P_0,\ ^3P_2$	$^3P_1,\ ^3P_2$	$^3P,\ ^5S^o$
N^+	2.64	0.29	0.83	0.41	0.27	1.12	1.27
O^{+2}	2.29	0.29	0.58	0.55	0.27	1.29	0.18
Ne^{+4}	2.09	0.25	0.58	1.41	1.81	5.83	1.51
Ne^{+2}	1.36	0.15	0.27	0.24	0.21	0.77	—
S^{+2}	6.95	1.18	1.38	3.98	1.31	7.87	2.85
Ar^{+4}	3.21	0.56	1.65	2.94	1.84	7.81	—
Ar^{+2}	4.83	0.84	1.22	1.26	0.67	3.09	—

(Osterbrock & Ferland, p. 53)

– calculated quantum-mechanically

Ex) Collision strength for [O III] $^3\mathrm{P} \rightarrow {}^1\mathrm{D} = 2.29$

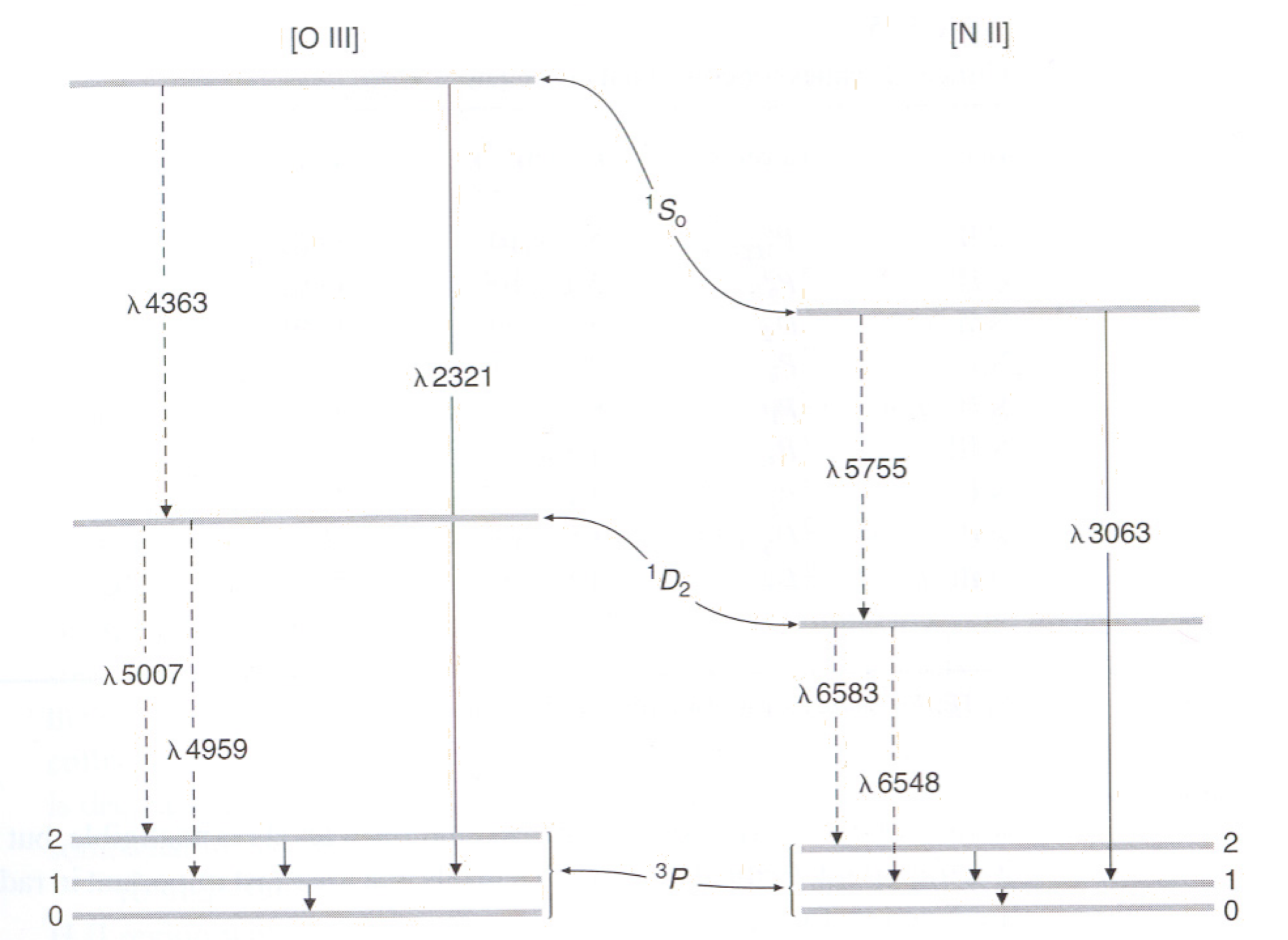
Radiative Transitions :

$^1\mathrm{D}_2 \rightarrow {}^3\mathrm{P}_2 :\ \lambda 5007$

$^1\mathrm{D}_2 \rightarrow {}^3\mathrm{P}_{\textcircled{1}} :\ \lambda 4959$

J = 1

Ex) Energy-Level Diagram for [O III], [N II]



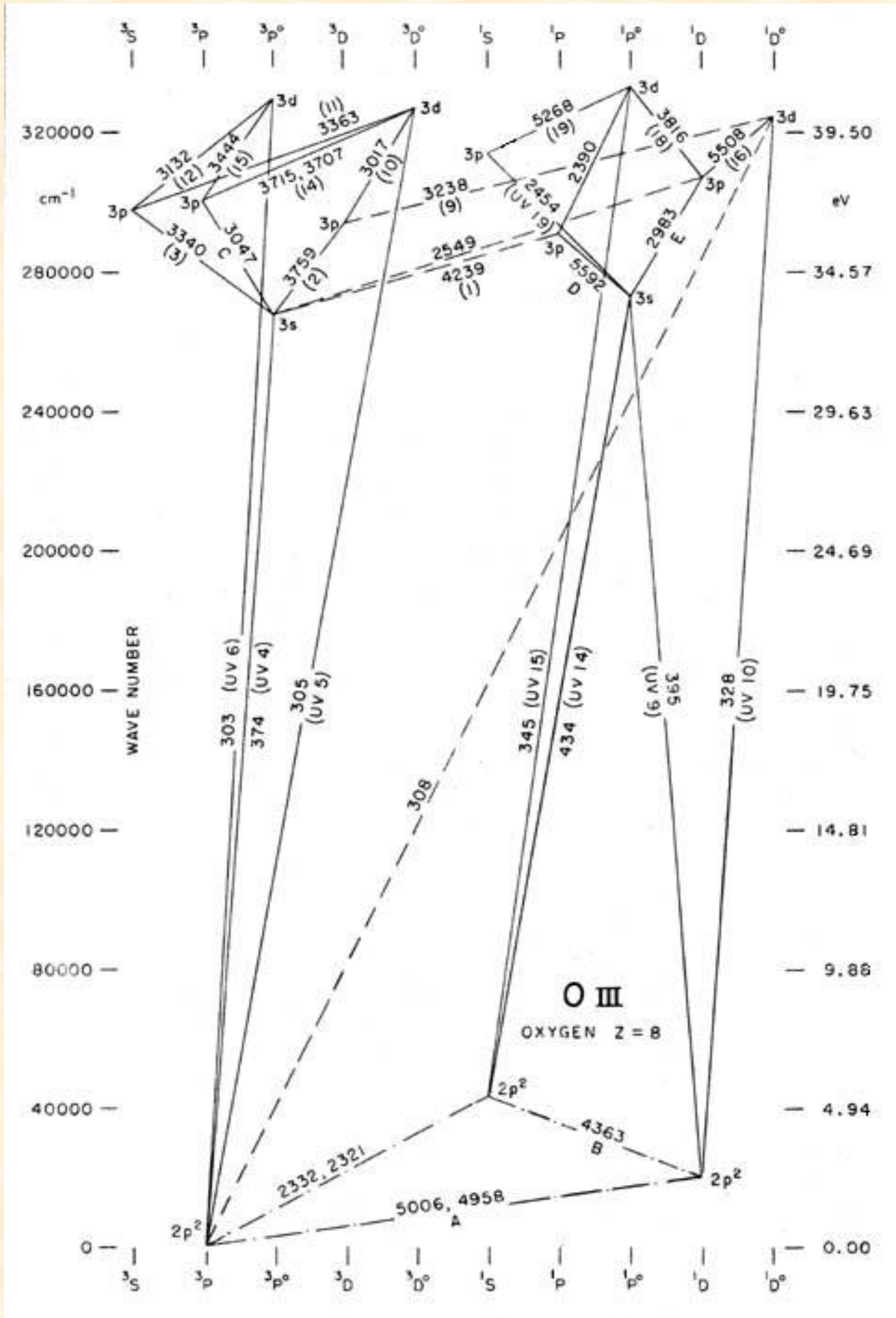
(Osterbrock & Ferland, p. 59)

Partial Grotrian Diagram for [O III]

O^{+2} (Carbon-like):
- Ground: 1s^{2}2s^{2}2p^2
- 2 outer shell electrons
- L-S coupling

from

Partial Grotrian Diagrams of Astrophysical Interest, Moore, C.E. & Merrill, P.W., NSRDS National Bureau of Standards, Vol. 23 (1968)



L =	0	1	1	2	2	0	1	1	2	2
S =	1	1	1	1	1	0	0	0	0	0

Collisional De-Excitation

- detailed balancing: populations of levels remain constant in equilibrium
- rate of population of a level = rate of depopulation
- relation between cross sections for collisional excitation and de-excitation can be derived from thermodynamic equilibrium (Osterbrock & Ferland, p. 50):

Consider a two-level transition (1- lower, 2 - upper):

$$\omega_1 v_1^2 \sigma_{12}(v_1) = \omega_2 v_2^2 \sigma_{21}(v_2)$$

where $\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + \chi$ (where $\chi = h\nu_{12}$)

So : $\sigma_{21}(v_2) = \dfrac{\pi\, h^2}{m^2 {v_2}^2} \dfrac{\Omega_{12}}{\omega_2}$ (similar to formula for $\sigma_{12}(v_1)$)

Collision Rates

The collisional de - excitation rate is:

de-excitations/vol/sec = $n_e n_2 q_{21}$

$$q_{21} = \int_0^\infty v\ \sigma_{21}\ f(v)\ dv \qquad (q_{21}\ \text{in cm}^3\text{s}^{-1})$$

(note similarity to recombination)

The collisional excitation rate is :

collisions / vol / sec $= n_e n_1 q_{12}$

where $q_{12} = \dfrac{\omega_2}{\omega_1} q_{21}\ e^{-\chi/kT}$

Note: q_{ij} is a function of (σ_{ij}, v) which is a function of

(Ω_{ij}, v) or (Ω_{ij}, T)

Energy Loss by Collisional Processes

1) Single excited level, low n_e

$\boxed{L_C = n_e n_1 q_{12} h\nu_{12}}$ (every excitation followed by radiative transition)

2) Single excited level, higher n_e

$$n_e n_1 q_{12} = n_e n_2 q_{21} + n_2 A_{21}$$

collisions/vol/sec = # de-excitations/vol/sec + # transitions/vol/sec

- solve above eqn. for $\frac{n_2}{n_1}$ to get relative level populations

Solve for population of level 2 :

$n_2 = n(X) - n_1 \quad n(X) = $ number density of element X

$$\boxed{L_c = n_2 A_{21} h\nu_{12}}$$

3) For multiple levels, use detailed balancing:
- multiple equations for each level, # in = # out

For each level i of an ion X :

$$\sum_{j\neq i} n_j n_e q_{ji} + \sum_{j>i} n_j A_{ji} = \sum_{j\neq i} n_i n_e q_{ij} + \sum_{j<i} n_i A_{ij}$$

(transitions into i) = (transitions out of i)

together with :

$$\sum_i n_i = n(X)$$

can be solved for the population in each level n_i.

$$\boxed{L_c = \sum_i n_i \sum_{j<i} A_{ij} h\nu_{ij}}$$

Critical Density

- For a given level i, n_c is the density at which
  # radiative transitions/vol/sec = # de-excitations/vol/sec

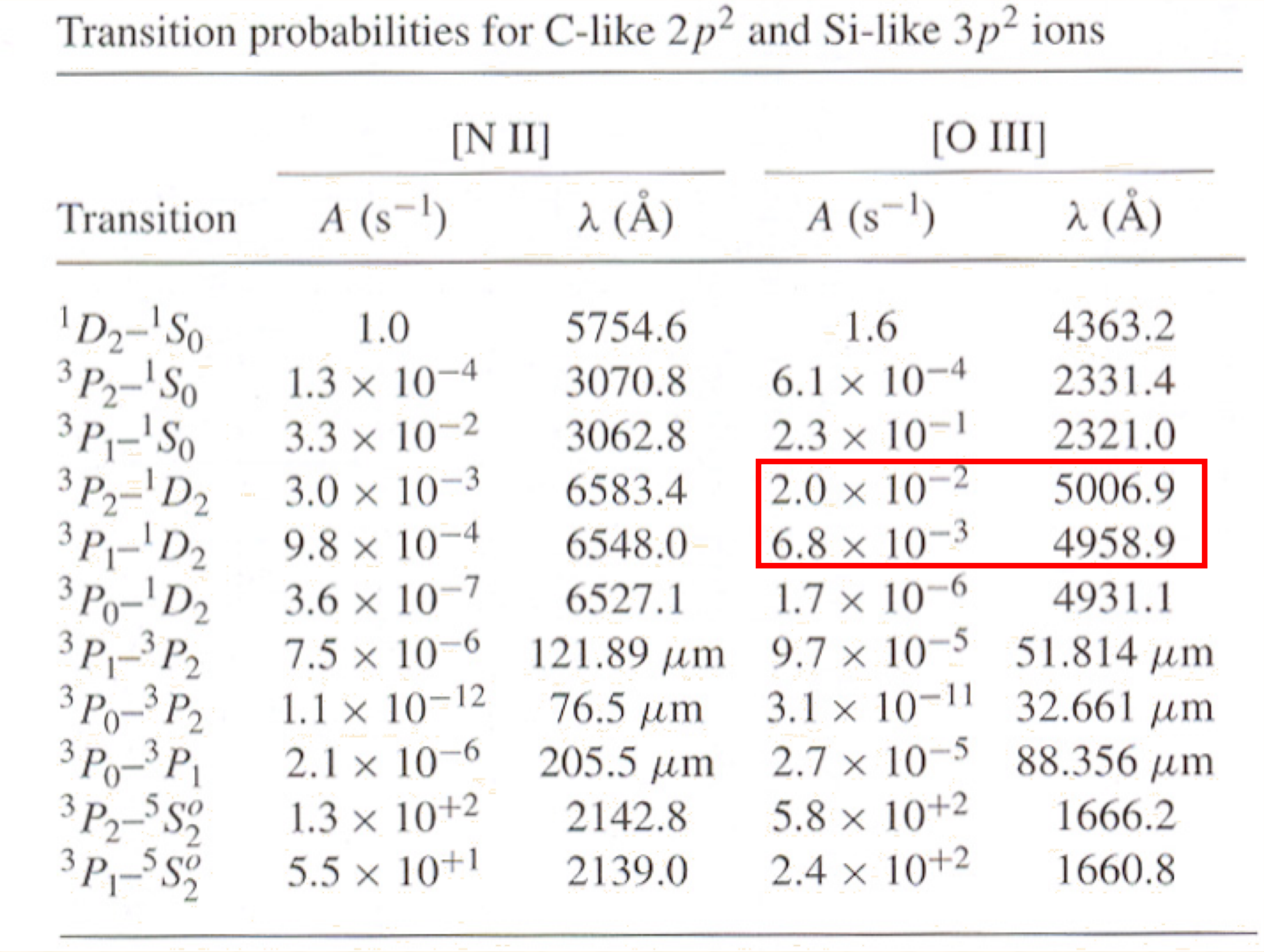
Let $n_e = n_c$ when this occurs:

$$n_i \sum_{j<i} A_{ij} = n_c n_i \sum_{j\neq i} q_{ij}$$

$$n_c(i) = \frac{\sum_{j<i} A_{ij}}{\sum_{j\neq i} q_{ij}}$$

-At densities $n_e > n_c$, line emission from $i \rightarrow j$ is significantly suppressed.

Transition Probabilities ("A" values)

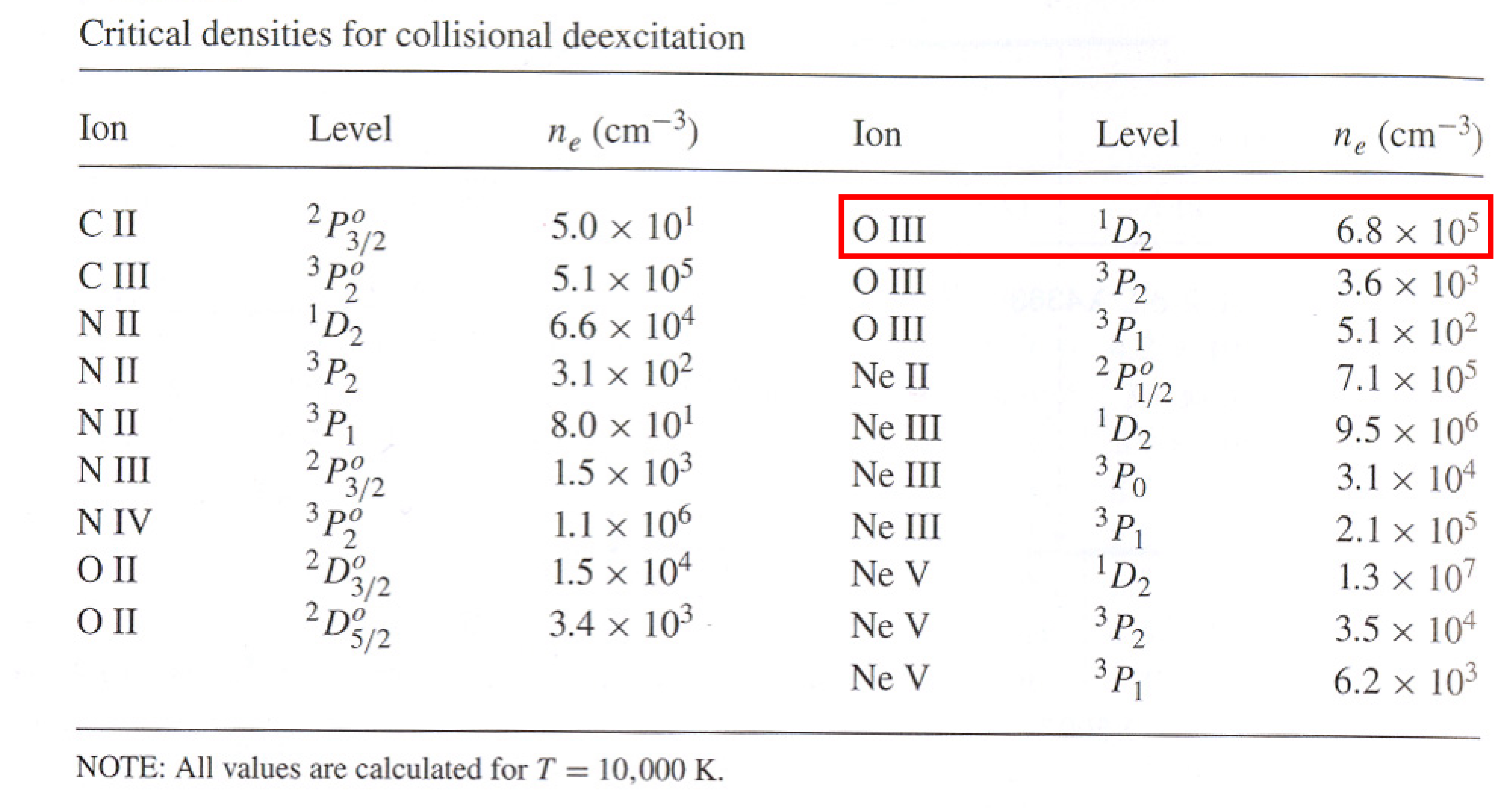


Transition probabilities for C-like $2p^2$ and Si-like $3p^2$ ions

	[N II]		[O III]	
Transition	A (s^{-1})	λ (Å)	A (s^{-1})	λ (Å)
$^1D_2$–$^1S_0$	1.0	5754.6	1.6	4363.2
$^3P_2$–$^1S_0$	1.3×10^{-4}	3070.8	6.1×10^{-4}	2331.4
$^3P_1$–$^1S_0$	3.3×10^{-2}	3062.8	2.3×10^{-1}	2321.0
$^3P_2$–$^1D_2$	3.0×10^{-3}	6583.4	2.0×10^{-2}	5006.9
$^3P_1$–$^1D_2$	9.8×10^{-4}	6548.0	6.8×10^{-3}	4958.9
$^3P_0$–$^1D_2$	3.6×10^{-7}	6527.1	1.7×10^{-6}	4931.1
$^3P_1$–$^3P_2$	7.5×10^{-6}	121.89 μm	9.7×10^{-5}	51.814 μm
$^3P_0$–$^3P_2$	1.1×10^{-12}	76.5 μm	3.1×10^{-11}	32.661 μm
$^3P_0$–$^3P_1$	2.1×10^{-6}	205.5 μm	2.7×10^{-5}	88.356 μm
$^3P_2$–$^5S_2^o$	$1.3 \times 10^{+2}$	2142.8	$5.8 \times 10^{+2}$	1666.2
$^3P_1$–$^5S_2^o$	$5.5 \times 10^{+1}$	2139.0	$2.4 \times 10^{+2}$	1660.8

(Osterbrock & Ferland, p. 56)

Critical Densities for Some Important Levels



Critical densities for collisional deexcitation

Ion	Level	n_e (cm^{-3})	Ion	Level	n_e (cm^{-3})
C II	${}^2P^o_{3/2}$	5.0×10^1	O III	${}^1D_2$	6.8×10^5
C III	${}^3P^o_2$	5.1×10^5	O III	${}^3P_2$	3.6×10^3
N II	${}^1D_2$	6.6×10^4	O III	${}^3P_1$	5.1×10^2
N II	${}^3P_2$	3.1×10^2	Ne II	${}^2P^o_{1/2}$	7.1×10^5
N II	${}^3P_1$	8.0×10^1	Ne III	${}^1D_2$	9.5×10^6
N III	${}^2P^o_{3/2}$	1.5×10^3	Ne III	${}^3P_0$	3.1×10^4
N IV	${}^3P^o_2$	1.1×10^6	Ne III	${}^3P_1$	2.1×10^5
O II	${}^2D^o_{3/2}$	1.5×10^4	Ne V	${}^1D_2$	1.3×10^7
O II	${}^2D^o_{5/2}$	3.4×10^3	Ne V	${}^3P_2$	3.5×10^4
			Ne V	${}^3P_1$	6.2×10^3

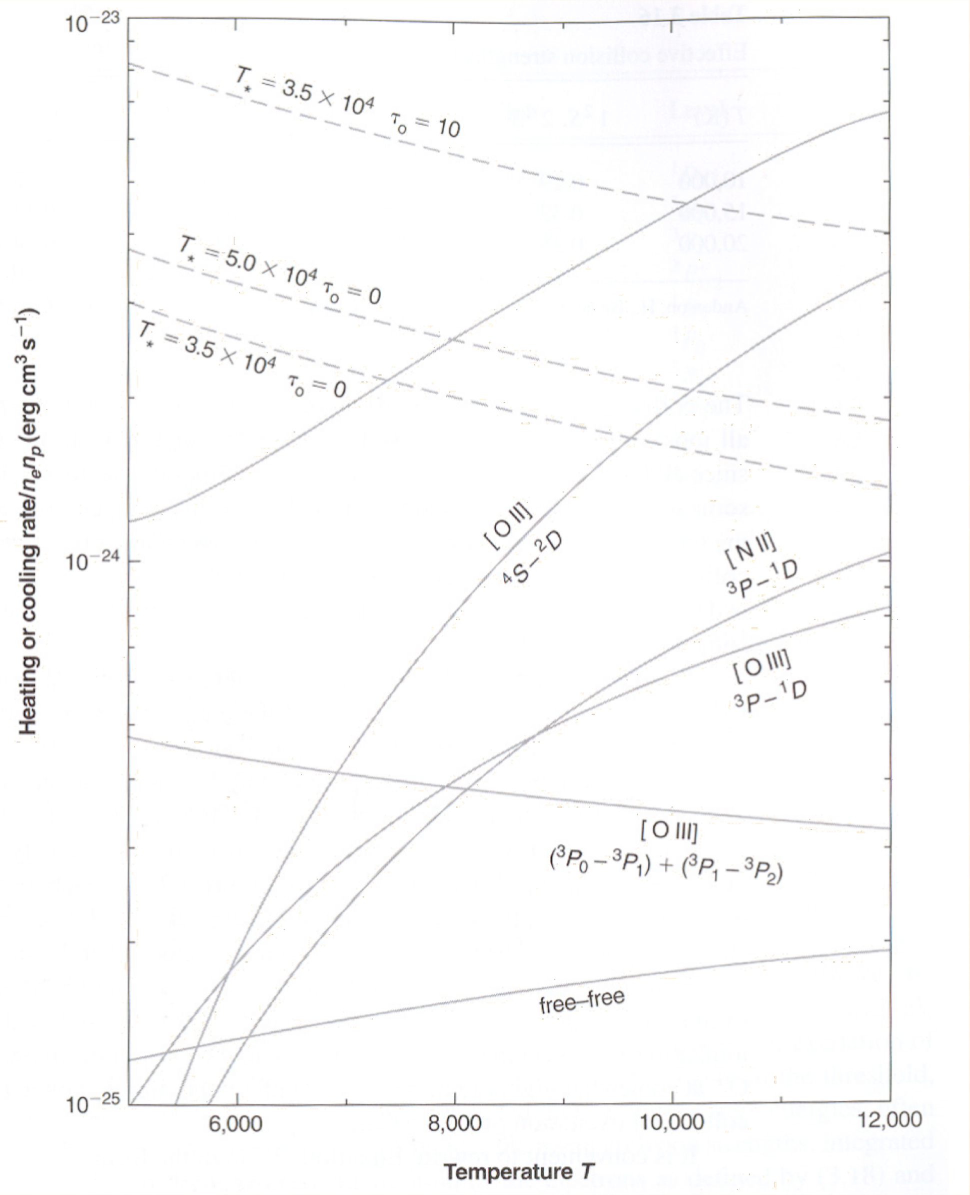
NOTE: All values are calculated for $T = 10{,}000$ K.

(Osterbrock & Ferland, p. 60)

Heating and Cooling Rates for a Low-Density Gas

effective heating
= cooling
$G - L_R = L_{ff} + L_c$

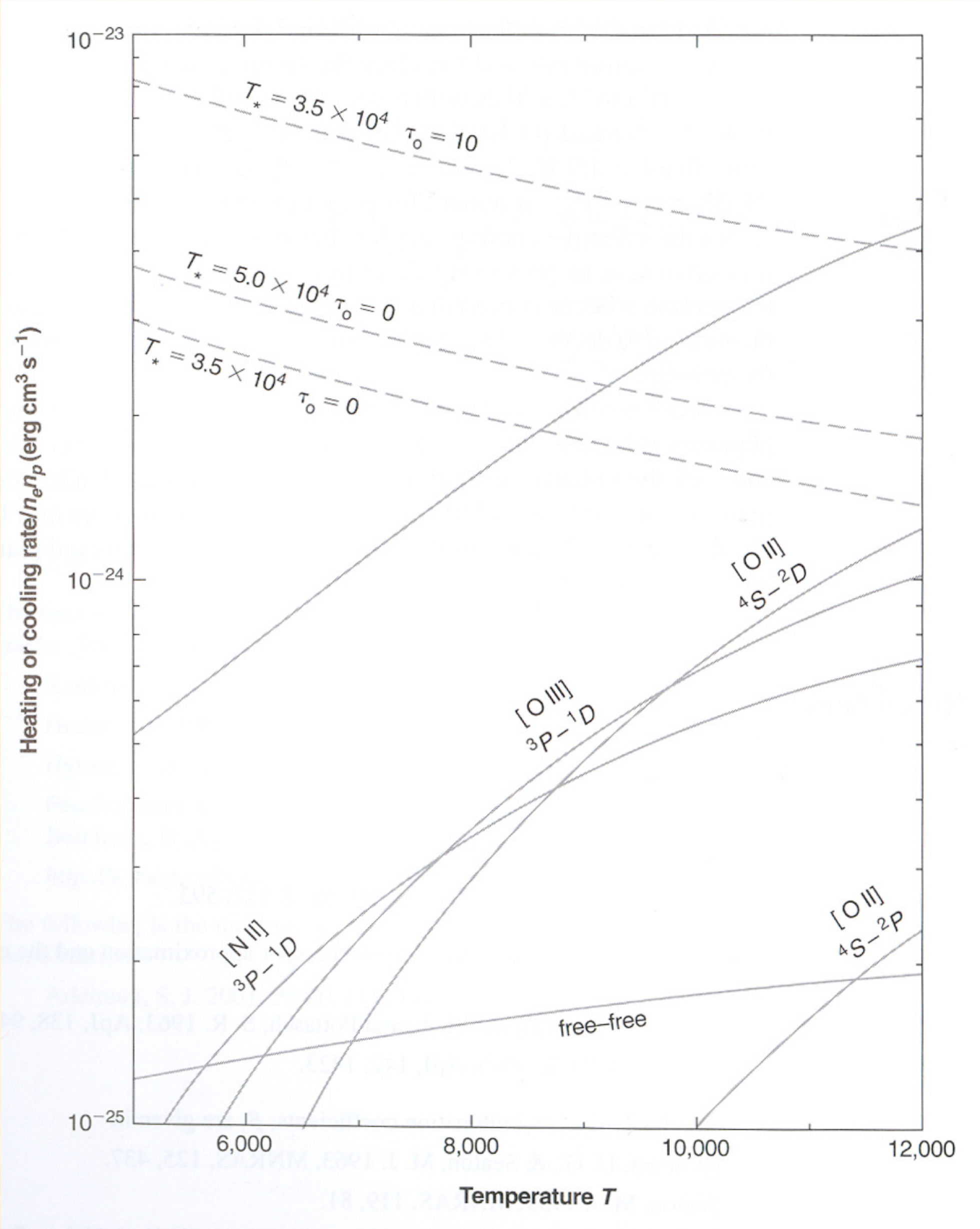
Per $n_e n_p$ -
$G - L_R$: dashed
$L_{ff} + L_c$: solid



(Osterbrock & Ferland, p. 62)

Heating and Cooling Rates for $n_e = 10^4 \mathrm{cm}^{-3}$

Collisional
de-excitation
raises temperatures



(Osterbrock & Ferland, p. 63)