Emission-Line Diagnostics

- Temperatures from collisionally excited lines
- Temperatures from recombination
- Densities from emission lines
- Ionizing spectrum from “photon counting”
  - The Zanstra method: temperature of ionizing star
- Abundances
Emission-Line Diagnostics - Summary

• Temperature Measurements
  - collisional excitation of two upper levels with very different excitation energies
  - comparison of recombination continuum and emission lines

• Density Measurements
  - excitation of two upper levels with similar energies, but different transition probabilities (different critical densities)

• Ionizing Radiation
  - use optically thick nebulae to count ionizing photons
  - presence of high-ionization lines to indicate “hardness” of ionizing radiation

• Abundances
  - when temperature and density are fixed, the remaining variable is abundances of the elements
Temperatures from Emission Lines

- Ex) [O III] $\lambda \lambda 4959, 5007$ arise from low $^1D_2$ level,
  
  [O III] $\lambda 4363$ from higher $^1S_0$ level:
- the ratio of $j(5007)/j(4959)$ is fixed at $3.0 = \text{ratio of radiative transition probabilities (since both from same upper level)}$
- as the temperature increases, the average electron velocity increases, which increases population of the $^1S_0$ level
- Thus, $j(4363)$ increases relative to $j(5007) + j(4959)$ as the temperature increases
- For low densities, the ratio depends only on temperature
- For densities of $n_e > 10^5$ cm$^{-3}$, $^1D_2$ begins to get collisionally de-excited
- Plugging in the atomic parameters (Osterbrock, chapter 5):

\[
\frac{j_{4959} + j_{5007}}{j_{4363}} = \frac{7.90 \exp \left[ (3.29 \times 10^4) / T \right]}{1 + 4.5 \times 10^{-4} \left( n_e / T^{1/2} \right)}
\]
Ex) Energy-Level Diagram for [O III], [N II]

(Osterbrock & Ferland, p. 59)
Ratios as Function of Temperature (Low Density)

(Osterbrock & Ferland, p. 110)
Temperatures for H II Regions

• Typical H II region temperatures are \(~10,000\) K
• Some disagreement from different diagnostics, which provide a good starting point, but real temperatures come from models.

(Osterbrock & Ferland, p. 112)
Temperatures for Planetary Nebulae

(Osterbrock & Ferland, p. 113)

- Higher temperatures than H II regions, on average
  - Hotter stars and higher densities (collision de-excitation decreases cooling efficiency)
- [O III] may be sampling higher ionization regions than [N II]
Temperature from Recombination Continuum, Lines

- Recombination lines are nearly independent of temperature, since they are dominated by cascades.
- Continuum flux is a function of temperature, since capture cross section decreases with increasing free electron velocity.
  1) Measure continuum flux on either side of the Balmer jump (3646 Å) or
  2) Measure HB emission-line flux and continuum flux nearby.

(Osterbrock & Ferland, p. 116)
Densities from Emission Lines

• Ex) [O II] \( \lambda \lambda 3726, 3729 \) are excited from ground level to two slightly different upper levels.

• The upper levels have different critical densities:
  \[ 2D_{3/2} = 1.6 \times 10^4 \text{ cm}^{-3} (\lambda 3726) \quad 2D_{5/2} = 3.1 \times 10^3 \text{ cm}^{-3} (\lambda 3729) \]
  - as density increases, \( j_{3729}/j_{3726} \) will decrease

• At zero density, \( j_{3729}/j_{3726} = 1.5 \) (ratio of statistical weights)
• At very high density, a Boltzman distribution is established:
  \[
  \frac{j_{3729}}{j_{3726}} = \frac{3}{2} \frac{A_{3729}}{A_{3726}} = \frac{3}{2} \frac{3.6 \times 10^{-5}}{1.8 \times 10^{-4}} \approx 0.34
  \]
  - [S II] - \( j_{6716}/j_{6731} \) works the same way
Ex) Energy-Level Diagram for [O II], [S II]

- Ground configuration $2p^3$ for [O II] and $3p^3$ for [S II]

(Osterbrock & Ferland, p. 122)
[O II], [S II] Ratio as Function of Density

(Osterbrock & Ferland, p. 123)
Densities for Planetary Nebulae

(Osterbrock & Ferland, p. 125)
Zanstra Method - Temperature of Ionizing Star

- Use the flux of nebular Hβ (F_{H\beta}) to count ionizing photons
- Measure the flux of the star (F_\nu) in the optical continuum near Hβ
- Use the ratio F_\nu / F_{H\beta} to obtain the temperature of the star

\# \text{ ionizations / sec} = \# \text{ recombinations / sec}

\[ Q(H^0) = \int_{v_0}^{\infty} \frac{L_\nu}{h \nu} \, dv = \int_0^r n_p n_e \alpha_B(H^0, T) \, dV \]

The total number of Hβ photons is:

\[ Q(H\beta) = \frac{L(H\beta)}{h \nu_{H\beta}} = \int_0^r j_{H\beta} \, dV = \int_0^r n_p n_e \alpha_{H\beta}^{\text{eff}}(H^0, T) \, dV \]

\[ \frac{Q(H\beta)}{Q(H^0)} \approx \frac{\alpha_{H\beta}^{\text{eff}}(H^0, T)}{\alpha_B(H^0, T)} \quad \text{so} \quad Q(H\beta) \propto Q(H^0) \]
To compare the luminosity of a star at any frequency $\nu$ with $Q(H^0)$:

$$\frac{L_\nu}{Q(H^0)} = \frac{L_\nu}{L_{H\beta} / \nu_{H\beta}} \frac{Q(H\beta)}{Q(H^0)} = h\nu_{H\beta} \frac{\alpha_{H\beta}^{\text{eff}}(H^0, T)}{\alpha_{B}(H^0, T)} F_{H\beta}$$

So this ratio depends primarily on the observed fluxes (you are counting $H\beta$ and nearby stellar continuum photons)

If we assume a blackbody distribution for $L_\nu$

$$\frac{L_\nu}{Q(H^0)}$$

can be tabulated for different temperatures

→ gives the temperature of the star (Zanstra method)

- more realistic determinations use stellar atmospheres
Abundances

- Once the temperature and density are known, a photoionization model can be calculated to get the emissivity of each line.
- In practice, this is an iterative process:
  1) calculate model
  2) adjust input parameters (ionizing spectrum and luminosity, density, geometry, etc.)
  3) compare observed and model line ratios (usually relative to Hβ)
  4) go back to step 1)

- For discrepant lines, you can adjust the abundances to get the proper ratios of C, N, O (etc.) lines.
- Beware: in practice, must account for reddening, density inhomogeneities, etc.
# Measured Abundances

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<th>N</th>
<th>Atom</th>
<th>Sun</th>
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<th>Planetary</th>
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*(Osterbrock & Ferland, p. 147)*