Gas Dynamics

- Gaseous nebulae are not static entities:
- Expansion of ionized gas into the ISM (Planetary Nebula)
- Radiation driving of ionized clouds (stellar winds and AGN)
- Gravitational motions around a supermassive black hole (AGN)
- Expansion of ionized “sphere” of gas after a hot star turns on (H II region)
  - creates an ionization/shock front
- Explosions into the surrounding ISM (novae and supernovae)
  - create shock fronts
Shock Fronts

Ex) Piston in a tube of gas (adiabatic case)

• Moving piston starts a compression wave (higher $P$, $\rho$).
• Speed of sound is higher in compressed region.
• Gas in the compressed region travels much faster, acting to steepen the pulse.
• A nearly discontinuous shock front is formed
• Note that velocity of piston must be supersonic (compared to unshocked gas), or the disturbance will spread out (no more shock front).
Shock Fronts

- Ex) Explosion of a Supernova into the ISM
- Supernova remnant moves at supersonic speed in the ISM at $v_0$
- Builds up a pulse of increased pressure
- Pulse steepens because sound velocity is higher in the compressed region. In this case, gas can escape out the back end.
- Consider a shock front propagating through the ISM:
  - Reference frame: traveling with the shock

\[
\begin{align*}
\rho_0, T_0 & \quad \text{ISM} \quad v_0 \quad \rho_1, T_1 \quad \text{shocked gas} \\
\rho &\quad \text{mass density, } T \quad \text{temperature, } v \quad \text{velocity}
\end{align*}
\]
Conservation laws

1) $\rho_0 v_0 = \rho_1 v_1$  (cons. of mass)

2) $P_0 + \rho_0 v_0^2 = P_1 + \rho_1 v_1^2$  (cons. of momentum)

I. Adiabatic Case (e.g., initial SNR explosion)

For adiabatic expansion (no radiation loss from compression):

$P = K \rho^\gamma$  ($\gamma = 5/3$ for monatomic gas)

"It can be shown that (see Osterbrock, p. 162)":

3) $\frac{1}{2} v_0^2 + \frac{\gamma}{\gamma - 1} \frac{P_0}{\rho_0} = \frac{1}{2} v_1^2 + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1}$  (cons. of energy)

$$\frac{1}{2} v_0^2 + \frac{5}{2} \frac{P_0}{\rho_0} = \frac{1}{2} v_1^2 + \frac{5}{2} \frac{P_1}{\rho_1}$$  (for monatomic gas)

(1st term - flow kinetic energy per mass, 2nd term - thermal kinetic energy plus compression energy)
II. Isothermal Case (e.g., stellar wind bubble inside H II region)
- gas just ahead and just behind shock has same temperature
  \( T_0 = T_1 \)
- applies to shocks within H I or H II regions since the
  heating and cooling time scales are much smaller than the
  expansion time scales (works also for planetary nebula)

\[
P = \frac{\rho kT}{\mu m_H} \quad (\mu \text{ - atomic weight, } T = \text{ const.})
\]

\[P \propto \rho \quad (\gamma = 1)\]

So for the conservation of energy:

3) \[
\frac{P_0}{\rho_0} = \frac{P_1}{\rho_1} = \frac{kT}{\mu m_H}
\]
The sound speed in the undisturbed gas is given by:

\[ c_0 = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma kT_0}{\mu_0 m_H}} \quad (\sim 1 \text{ km s}^{-1} \text{ for } T = 100 \text{ K}) \]

and the Mach number of the shock front is:

\[ M = \frac{|v_0|}{c_0} \quad \text{Supersonic if } M > 1 \]

Combining the conservation equations, we get:

1) \[ \frac{P_1}{P_0} = \frac{2\gamma}{\gamma + 1} M^2 - \frac{\gamma - 1}{\gamma + 1} \]

2) \[ \frac{\rho_1}{\rho_0} = \frac{(\gamma + 1)M^2}{(\gamma - 1)M^2 + 2} \]

Ex) For a strong shock (large M), \( P_1 / P_0 \rightarrow \infty \)

Adiabatic monatomic case: \( \rho_1 / \rho_0 \rightarrow 4, \quad v_1 \rightarrow \frac{1}{4} v_0 \)

Isothermal case: \( \rho_1 / \rho_0 \rightarrow \infty, \quad v_1 \rightarrow 0 \)
Ionization Fronts

• Ex) An O or B star turns on
• An H II region expands at the rate that ISM is ionized
• The momentum conservation law is the same. The degree of ionization changes sharply across the boundary:

1) \( \rho_0 v_0 = \rho_1 v_1 = m_i \Phi_i \) (cons. of mass)

where \( m_i \) = mass of ionized gas per created ion / electron pair

\( m_i = m_H \) for a pure H nebula

\( \Phi_i = \int_{v_0}^{\infty} \frac{F_v}{h\nu} \, dv = \text{flux of ionizing photons} \)
Ionization Fronts (H II Region)

For an ionized nebula, we have two very different temperatures. The conservation of energy becomes:

\[
3) \quad \frac{P_0}{\rho_0} = \frac{kT_0}{\mu_0 m_H} \quad \text{and} \quad \frac{P_1}{\rho_1} = \frac{kT_1}{\mu_1 m_H} \quad (T_1 \text{ fixed by photoionization,} \quad T_0 \text{ is temperature of ISM})
\]

\[
\rho_0, T_0 \quad \rightarrow \quad \rho_1, T_1
\]

ISM \quad v_0 \quad \rightarrow \quad \text{Ionized gas} \quad v_1

Solving for the density ratio:

\[
\frac{\rho_1}{\rho_0} = \frac{c_0^2 + v_0^2 \pm \left[\left(c_0^2 + v_0^2\right)^2 - 4c_1^2v_0^2\right]^{1/2}}{2c_1^2} \quad \text{(Osterbrock, p. 166)}
\]
There are two allowed ranges for \( v_0 \) (speed of ionization front),

since \( \frac{\rho_1}{\rho_0} \) must be real:

1) \( v_0 \geq c_1 + \sqrt{c_1^2 - c_0^2} \equiv v_R \approx 2c_1 \) (for \( c_1 \gg c_0 \) in H II region)

\( v_R \) is the velocity of an "R-critical front" (R - rare or low density),

since as \( \rho_0 \to 0, \ v_0 \to \infty \) and therefore exceeds \( v_R \)

For \( v_0 > v_R \), an R-type front moves supersonically into the ISM

2) \( v_0 \leq c_1 - \sqrt{c_1^2 - c_0^2} \equiv v_D \approx \frac{c_0^2}{2c_1} \) (\( c_1 \gg c_0 \))

\( v_D \) is the velocity of a "D-critical front" (D - dense),

When \( v_0 < v_D \), a D-type front moves subsonically into the ISM
Consider an H II region expanding into the ISM

In this case, \(c_0\) is small, and \(v_0 \gg c_1\), so again there are two cases:

1) \(\frac{\rho_1}{\rho_0} = \frac{v_0^2}{c_1^2} \left(1 - \frac{c_1^2}{v_0^2}\right) \gg 1\)  (strong R-type front)

2) \(\frac{\rho_1}{\rho_0} = 1 + \frac{c_1^2}{v_0^2} \approx 1\)  (weak R-type front)

Case 1) is not physical, since high densities in the ionized gas disrupt the shock. Using the conservation of mass for case 2):

\[v_1 = v_0 \left(1 - \frac{c_1^2}{v_0^2}\right) \approx v_0 \gg c_1\]

So the ionization front initially moves supersonically into the ISM, due to the large number of ionizing photons.
Model of Expanding H II Region around an O Star

- \( t = 0 - 300 \) years: weak R-type ionization front moves out at \( v_0 \approx 300 \) km/sec
- \( t = 300 - 4600 \): \( \Phi_i \) decreases as the front expands, due to geometric dilution and photoionization; \( v_0 \) decreases until it reaches \( v_R \approx 2c_1 \)
- \( t \approx 4600 \) years: a shock front breaks off and compresses the gas ahead of the ionization front, which becomes D-type; \( v_0 \approx v_D \), \( v_1 \approx c_1 \)
- \( t = 4600 - 1 \) million years: the ionization front continues as D-type, which slows down until it terminates at the Stromgren radius.

(Osterbrock & Ferland, p. 167)