THE BAADE-WESSELINK METHOD

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IMAGE CREDIT: NASA/STSCI
Moving Cluster
Cluster HRD fitting
Baade-Wesselink

Works from 133 pc - 50 Kpc
CEPHEIDS: A REMINDER

Variable Star

[Diagram showing the brightness of a variable star over time]

brightness

time
DISTANCE TO CEPHEIDS

- Cepheids: rare, luminous, regular variation
- Period-Luminosity relationship: useful, needs good zero-point calibration!
- Uncertainty: 22-34%
- Need independent measures of distances to calibrate.
CLASSICAL BAADE-WESSELINK METHOD

• Get speed of expansion & time for expansion, calculate physical distance.

• Photosphere approximated as spherically symmetric shell.

• Integral of velocity gives physical change in radius.

\[ \Delta R = R(t) - R(t_0) = \int_{t_0}^{t} v(t) \, dt \]

• If you have the change in angular diameter, you can calculate distance:

\[ d = \frac{2\Delta R}{\Delta \Theta} \]

• Measure \( \Delta R \) from spectroscopy (radial velocity), measure \( \Delta \Theta \) from surface-brightness relation
CLASSICAL BAADE-WESSELINK METHOD

- Measure the color of the star at the minimum and maximum light to get surface brightness
- Measured in V & K bands (V band less sensitive to metallicity & shocks)
CLASSICAL BAADE-WESSELINK METHOD

• Compare surface brightness to apparent magnitude at those points —> gives change in angular diameter, $\Delta \Theta$

• Tells you about how long the expanding shell took to move

Observational surface brightness / color relationship
CLASSICAL BAADE-WESELINK METHOD

- Surface Brightness from color: $SB_{mag} = 2.656 + 1.483(V - K) + 0.044(V - K)^2$

- Angular Diameter: $\Theta_{mas} = 10^{0.2(SB-V)}$

- Now you know how far the shell moved ($\Delta R$) and how the angular size changed (relative to the time it took — $\Delta\Theta$)

- Combine these two to get physical distance to the star: $d_{pc} = 9.305 \frac{\Delta R(R_\odot)}{\Delta\Theta_{mas}}$
GEOMETRIC BAANDE-WESSELINK METHOD

- Improvement: measure angular size change with interferometers instead of surface brightness relationship.
- Requires very long baselines — largest angular size = 3 mas
- Only feasible for closest pulsators

$$d \ [\text{pc}] = 9.305 \ \Delta R \left[ R_\odot \right] / \Delta \theta \ [\text{mas}]$$
COMPLICATIONS

• Observed radial velocity from spectral lines is projected, isn’t equal to radial expansion velocity.

• Projection factor between observed and actual must be included (complicated):
  \[ v(t) = p \times v_{rad}(t) \]

• Determined from models, \( p \sim 1.3 - 1.5 \)

• Also, complications from likely velocity gradients in photosphere from movements of matter
CLASSICAL VS. GEOMETRIC

- Points: Interferometric observations
- Line: Surface brightness relation
- Good agreement!

BAADE-WESSELINK STEPS SUMMARY

- Obtain photometric V & K measurements at minimum & maximum brightness OR obtain interferometric observations of pulsator.

- Use these measurements to get the change in angular size (from surface brightness relation, or directly), $\Delta \Theta$.

- Obtain spectroscopic RV measurements.

- Use RV measurements along with P-factor to get change in physical radius, $\Delta R$.

- Calculate physical distance $D$ from change in angular size $\Delta \Theta$ and change in physical size $\Delta R$.

- Use physical distance and apparent magnitude to get intrinsic luminosity, $L$. 
BAADE-WESSELINK ERROR SOURCES

• Without interferometry — Photometric measurements: ~10%

• With interferometry — angular size (Θ) measurement: ~14%

• Limb darkening: ~2%

• P-factor uncertainty: ~3%

• RV resolution: ~8%

• Magnitude scale zero-point: ~5%

• Absolute flux calibration: ~ 5%

Total uncertainty: 6-14%

Period-Luminosity uncertainty: 22-34%

- Gaia provided distances to 300 Cepheids with a 3% error.
- If distance is known, p-factor can be derived.
- Constraining relationship between period & p-factor.
BAADE-WESSELINK CURRENT RESULTS

• Generalization of surface brightness, using effective temperature variations to determine Cepheid parameters (distance, radius, luminosity, color excess, intrinsic color) — Yaroslav et al. 2019

• New near-IR period-luminosity-relation for RR Lyrae based on Baade-Wesselink analysis — Muraveva et al. 2015

• Further constrained p-factor & period relationship, speculation about metallicity dependence of p-factor — Groenewegen, 2013
SUMMARY

• Measure change in angular size & change in physical radius to get distance.

• Surface brightness can go out to 50 kpc, interferometric can go out to 1 kpc

• Difficult (requires spectroscopy, photometry, and/or interferometry) but more precise than P-L relationship

• Used to calibrate P-L relationship
REFERENCES

• Wesselink, A., 1946, Bulletin of the Astronomical Institutes of the Netherlands. 10: 91–100
• https://www.eso.org/public/news/eso0432/