1. a) \( 1 + z = \frac{1 + \beta}{\sqrt{1 - \beta}} \rightarrow (1 + z)^2 = \frac{1 + \beta}{1 - \beta} \)

\( (1 + \beta) = (1 + z)^2 (1 - \beta) = (1 + z)^2 - (1 + z)^2 \beta \)

\( \beta + \beta (1 + z)^2 = (1 + z)^2 - 1 \)

\( \beta = \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1} \)

b) You can solve this algebraically or graphically. For a 1% error.

\( \frac{z - \beta}{z} = 0.01 \) so \( \frac{\beta}{z} = 0.99 \)

\( \frac{1}{z} \frac{(1 + z)^2 - 1}{(1 + z)^2} = 0.99 \)

\( \frac{z^2 + 2z}{z^2 + 2z + 2} = 0.99z \)

\( 0.99z^3 + 2(0.99)z^2 + (2)(0.99)z = z^2 + 2z \)

\( 0.99z^2 + 1.98z + 1.98 = z + 2 \)

\( 0.99z^2 + 0.98z - 0.02 = 0 \)

Using the quadratic equation:

\( z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

\( z = 0.02 \) (the other solution is -1.01, which is not valid)

For a 10% error, use the same formalism to get:

\( z = 0.2 \) (the other solution is -1.09, which is not valid)
c) The Hubble flow velocity would have to be ~6000 km s\(^{-1}\). 
\[ z = \frac{v}{c} = 0.02, \quad \text{Galaxies with } z < 0.02 \text{ are affected most severely.} \]

2. a) NGC 4151: RA = 12h10m32.6s +39d24m21s (J2000) 
b) \( z = 0.003319 \), \( cz = 995 \text{ km/s (heliocentric)} \) 
- from H I 21-cm emission from host galaxy, Third Reference Catalog of Bright Galaxies (RC3), deVaucouleurs et al. (1991)
c) Distance from \( z \): \( D = \frac{cz}{H_0} = 13.6 \text{ Mpc} \), where \( H_0 = 73 \text{ km/sec/Mpc} \) 
Corrected \( z \) to CMB frame: \( D = 17.0 \text{ Mpc} \) 
Distance from other techniques (average) = 14.1 Mpc 
- huge variation depending on technique!
d) R’SAB(rs)ab – Spiral galaxy with weak bar, outer pseudo-ring, inner ring/spiral, early type (deVaucouleurs et al. 1991)
Seyfert 1.5
e) \[ \theta = \frac{r}{D} = \frac{1 \text{ kpc}}{13.6 \times 10^3 \text{ kpc}} = 7.35 \times 10^{-5} \text{ rad} \left( \frac{206,265 \text{ arcsec}}{1 \text{ rad}} \right) = 15 \text{ arcsec} \] 
Scale = 15 arcsec/kpc

From NED: Scale = 12 arcsec/kpc (using CMB correction)
f) Major, minor axes = 6.3 x 4.5 arcmin, 31 x 22 kpc (from POSS)
g) \( \text{Inc} = \cos^{-1}(a/b) = 44^\circ \)

3. a) Assume you are looking at a torus from different inclination angles along a hemisphere. At a given distance (\( r \)), the probability of viewing at angle \( \theta \) is proportional to the solid angle determined by an annulus perpendicular to the disk with width \( d\theta \). So: 
\[ dP(\theta) = \sin(\theta)d\theta \] (probability increases with increasing \( \theta \))

The probability observing between angles \( \theta_1 \) and \( \theta_2 \) is:
\[ P = \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta \int_{\theta_1}^{\pi/2} \sin \theta \, d\theta \]
\[ P = \cos(\theta_1) - \cos(\theta_2) \]

The probability of observing a certain ratio of Seyfert 2s to Seyfert 1s is:
4. a) Sample biases:
1) Eddington bias: random errors in AGN magnitudes tend to increase number counts in surveys to a certain flux limit
2) Variablity: variable sources may be above or below the flux limit at any specific time
3) Emission-line equivalent width: surveys based on emission lines may miss sources with weak emission lines (low equivalent widths)
4) Absorption lines: strong absorption lines may suppress continuum flux, especially shortward of Lyα
5) Internal absorption: reddening by internal dust can suppress the entire flux, especially in the blue; strongly affects surveys that use color selection

b) Biases in specific wavebands
1) Radio: miss radio-quiet AGN, 90% of the population
2) Infrared: could miss Type 2 AGN in near-IR (not seeing inner part of torus, confusion with intense starbursts in far-IR
3) Optical/UV: miss AGN that are heavily reddened/extincted by host galaxy; miss some Type 2 AGN obscured by torus
4) EUV: miss about everything!
5) X-rays: miss obscured (Type 2) AGN in soft X-rays; miss some Compton-thick AGN even in hard X-rays

\[
x = \frac{\text{# Seyfert 2s}}{\text{# Seyfert 1s}} = \frac{N_2}{N_1}
\]

\[
P = \frac{N_2}{N_1 + N_2} \Rightarrow P = \frac{x}{x+1}
\]

\[
P = \cos(\theta_{\text{min}}) - \cos(90) = \cos(\theta_{\text{min}})
\]

\[
\theta_{\text{min}} = \cos^{-1}(P) = \cos^{-1}\left(\frac{x}{x+1}\right)
\]

For \(x = 1\), \(\theta_{\text{min}} = 60°\)
For \(x = 2\), \(\theta_{\text{min}} = 48°\)
For \(x = 3\), \(\theta_{\text{min}} = 41°\)