ASTRONOMY 8300 – Fall 2024
Homework Set 1 Answers

1.a. Maxwellian distr: $P(\mathrm{v}_r) = \frac{1}{\sqrt{\pi}b} e^{-(\mathrm{v}_r/b)^2}$

Gaussian profile: $P(\mathrm{v}_r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\mathrm{v}_r/\sigma)^2} \rightarrow b = \sqrt{2}\sigma$

$P(0) = \frac{1}{\sqrt{2\pi}\sigma}$ (amplitude of Gaussian)

$P\ (HWHM) = \frac{1}{2}P(0) = \frac{1}{2}\frac{1}{\sqrt{2\pi}\sigma} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(HWHM/\sigma)^2}$

$\frac{1}{2} = e^{-\frac{1}{2}(\mathrm{v}_r/\sigma)^2} \rightarrow \ln\left(\frac{1}{2}\right) = -\frac{1}{2}\left(HWHM/\sigma\right)^2$

$HWHM = \sqrt{2\ln(2)}\sigma = 1.1774\sigma$

$FWHM = 2HWHM = 2.355\sigma$

1.b. $b = \sqrt{\frac{2kT}{m}} = 0.17\ \mathrm{km\ s^{-1}}$ where $m = (28.09)(1.67\times10^{-24}\,\mathrm{g})$

$\sigma = \frac{b}{\sqrt{2}} = 0.12\ \mathrm{km\ s^{-1}}$

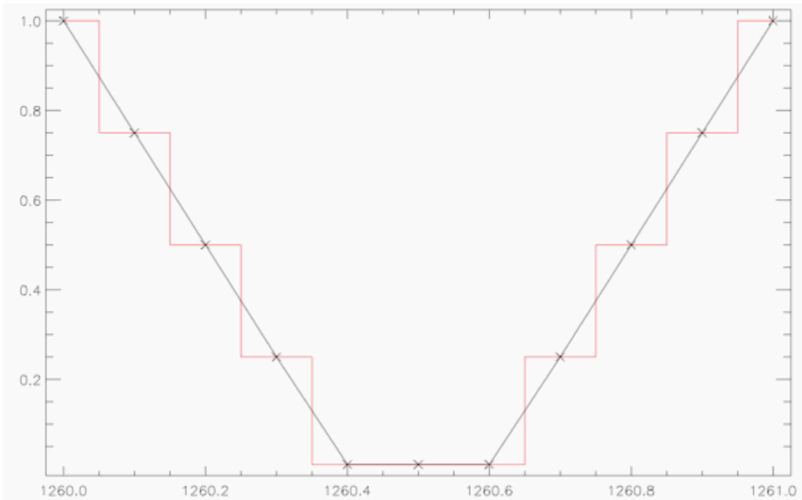
$FWHM = 2.355\sigma = 0.28\ \mathrm{km\ s^{-1}}$

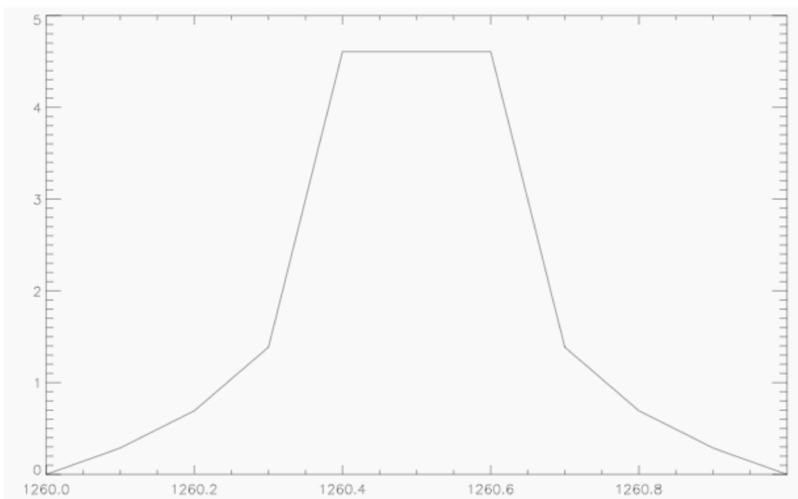
1.c. In radial velocity: $FWHM(LSF) = \dfrac{\Delta\lambda}{\lambda}\mathrm{c} = 11.9\ \mathrm{km\ s^{-1}} \rightarrow \mathrm{no}$

1.d. Linear part of the curve of growth for one line (lower limit to column density), full COG for more than one line: FWHM(line) << FWHM(LSF) (lines are unresolved)

1.e. Optical depth method: FWHM(line) > FHWM(LSF) (line is resolved)

2. Absorption-line (F_λ/F_c) and optical depth (τ = - ln [F_λ/F_c]) profiles:





2.a. $FWHM = \frac{\Delta\lambda}{\lambda}\mathrm{c} = \frac{0.6\text{Å}}{1260.4\text{Å}}(3.0\times10^5\ \mathrm{km\ s^{-1}}) = 143\ \mathrm{km\ s^{-1}}$

$\mathrm{v}_r = \frac{\Delta\lambda}{\lambda}\mathrm{c} = \frac{0.1\text{Å}}{1260.4\text{Å}}(3.0\times10^5\ \mathrm{km\ s^{-1}}) = 24\ \mathrm{km\ s^{-1}}$

2.b. $W_\lambda = \int\left(1 - F_\lambda/F_c\right)\,\mathrm{d}\lambda = 0.594\ \text{Å}$

2.c. $N = 1.1298\times10^{20}\frac{1}{f\lambda^2}W_\lambda = \frac{1.1298\times10^{20}}{(0.959)(1260.4)^2}(0.594) = 4.4\times10^{13}\mathrm{cm^{-2}}$

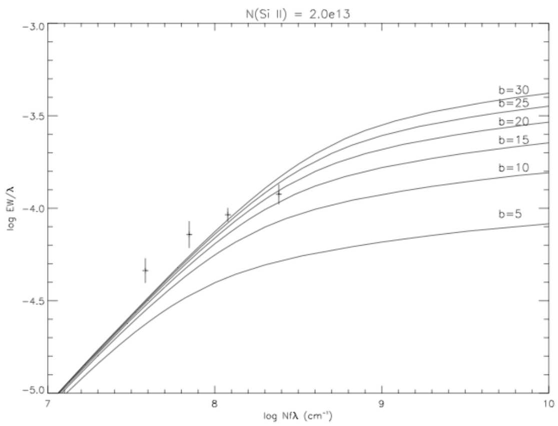
2.d. $N = 1.1298\times10^{20}\frac{1}{f\lambda^2}\int\tau_\lambda\ \mathrm{d}\lambda = \frac{1}{(0.959)(1260.4)^2}(1.85) = 1.37\times10^{14}\mathrm{cm^{-2}}$

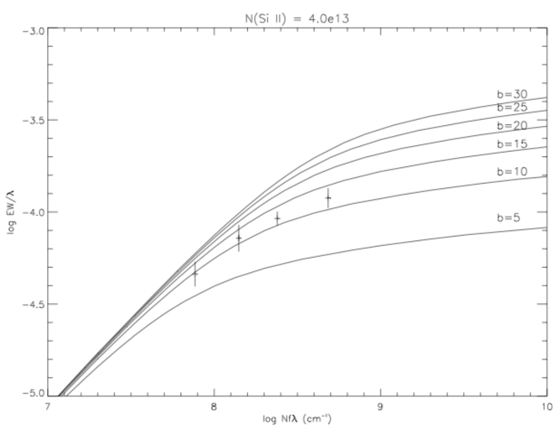
3. Use oscillator strengths (f) from Morton et al. (1991). Determine W_λ/λ and $Nf\lambda$ as a function of τ_0 for different values of b to get curves of growth. Note for the equation:

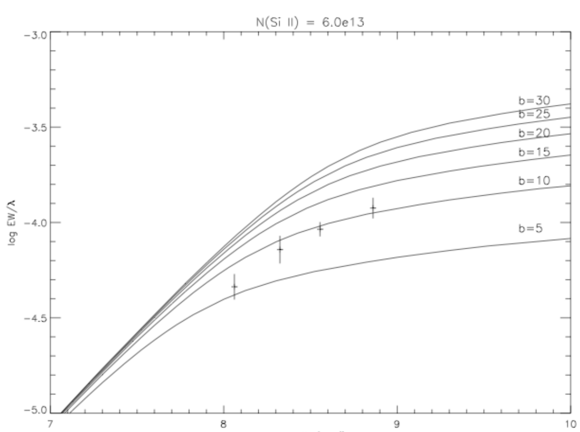
$$\tau_0 = \frac{N_j s \lambda_{jk}}{\sqrt{\pi} b} = \frac{1.497 \text{ x } 10^{-2}}{b} N_j \lambda_{jk} f_{jk},$$

the parameters are in cgs units, so b should be in cm/sec (see Spitzer, Ch. 3). Assume different columns to get $Nf\lambda$ for data points and overplot on curves of growth:

Answer: N(Si II) = 4 (± 1) x 10^{13} cm^{-2}, b = 12 (± 2) km s^{-1}







Propagation of Errors (Bevington 1969):

$$y = \log\left(\frac{W_\lambda}{\lambda}\right) = \frac{1}{2.3}\ln\left(\frac{W_\lambda}{\lambda}\right)$$

$$\sigma_y = \frac{1}{2.3}\left(\frac{\sigma_{W_\lambda}}{W_\lambda}\right)$$