ASTRONOMY 8300 – Fall 2024 Homework Set 3, due 10/21/20 at 9:30 AM

1 a. (5 pts) Given an O star with an effective temperature of $T_e = 50,000$ K and a radius of $R = 12.0 \text{ R}_{\odot}$, what is the O star's luminosity (assuming a blackbody)? What is the luminosity of a B star with $T_e = 30,000$ K and R = 7.5 R \odot ?

b. (10 pts) What are the above O and B star luminosities in ionizing flux ($hv \ge 13.6$ eV)? How many ionizing photons are the O and B stars each emitting per second?

c. (10 pts) Assuming a spherical hydrogen nebula with a density of $n_e = 10 \text{ cm}^{-3}$, what are the Strömgren radii for the O and B stars? How many B stars would be needed to ionize the same volume as a single O star?

2. Suppose you have a pure H nebula with a temperature of T = 10,000 K and density of $n_{\rm H} = 10$ cm⁻³ around the O star above. Assume Case B recombination applies.

a. (10 pts) What is the ionization time scale (t_{ion} , the length of time between ionizations of a neutral hydrogen atom) at a distance of 5 pc, assuming no significant absorption of photons at smaller distances from the star?

b. (10 pts) What is the recombination time scale per atom?

c. (10 pts) What is the ratio of ionized to neutral atoms n_p/n_{H0} in terms of the above time scales? Why are the above time scales so different?

3. Suppose that you want to build a simple model of a pure H nebula around a single star, by assuming constant temperature of the gas (T = 10,000 K), constant hydrogen density ($n_H = 10 \text{ cm}^{-3}$), and Case B recombination. Your task is to determine the ionization fraction (n_p/n_H) as a function of distance from the O star given above (similar to that shown in Osterbrock & Ferland, p. 26).

a. (20 pts) Give the equations that you would use in your program in terms of known or derived quantities. Give a brief step-by-step procedure of how you would work out the solution in your code.

b. (25 pts): Solve this problem. Warning: this requires careful programming and high-precision computing.