1. Suppose that you had a face-on disk galaxy at a distance of 10 Mpc that contains only solar-type stars (and no dust), with a uniform luminosity per area of $1 \, \text{L}_\odot \, \text{pc}^{-2}$.

a) What is the surface brightness of this galaxy in the B band ($\mu_B$) in mag arcsec$^{-2}$?

b) If there was an identical face-on galaxy behind it at a distance of 50 Mpc, what would the combined surface brightness ($\mu_B$) be in the area of overlap?

c) What if the more distant galaxy was inclined to the line of sight by $60^\circ$? What would the combined surface brightness ($\mu_B$) be in the area of overlap?

d) If the diameter of each galaxy in c) was 30 kpc, what would be their separate and combined B magnitudes?

2. Given the Schechter luminosity function for galaxies, what is the average luminosity density (in $\text{L}_\odot \, \text{Mpc}^{-3}$) of the present-day Universe? If the critical density for the present-day Universe is $2.8 \times 10^{11} \, h^2 \, \text{M}_\odot \, \text{Mpc}^{-3}$, what would the corresponding $M/L$ (mass-to-light ratio in solar units) be for a critical-density Universe?

3. Surface brightness profiles and luminosity:

a) Show that the deVaucouleur $R^{1/4}$ law results in a total luminosity of $7.22 \, \pi \, R_e^2 \, I(R_e)$.

b) Show that half of the above luminosity come from within a radius $R_e$.

c) Derive an expression for the total luminosity in a disk galaxy, where the surface brightness along the major axis is characterized by the exponential law.