EXTRAGALACTIC ASTRONOMY

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Secular and statistical parallax

Presenter

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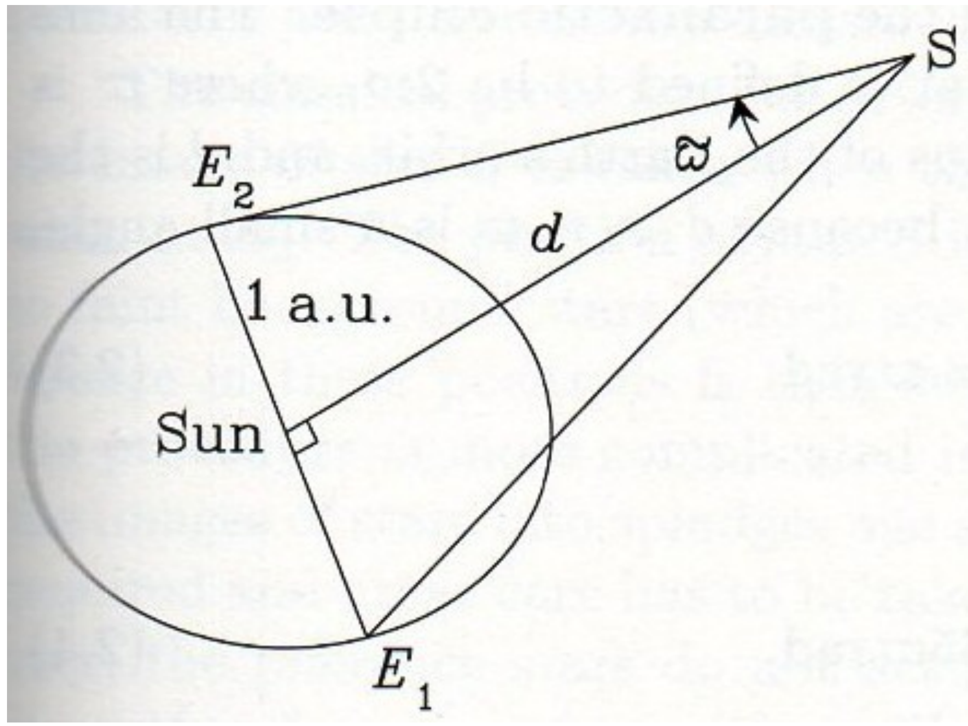
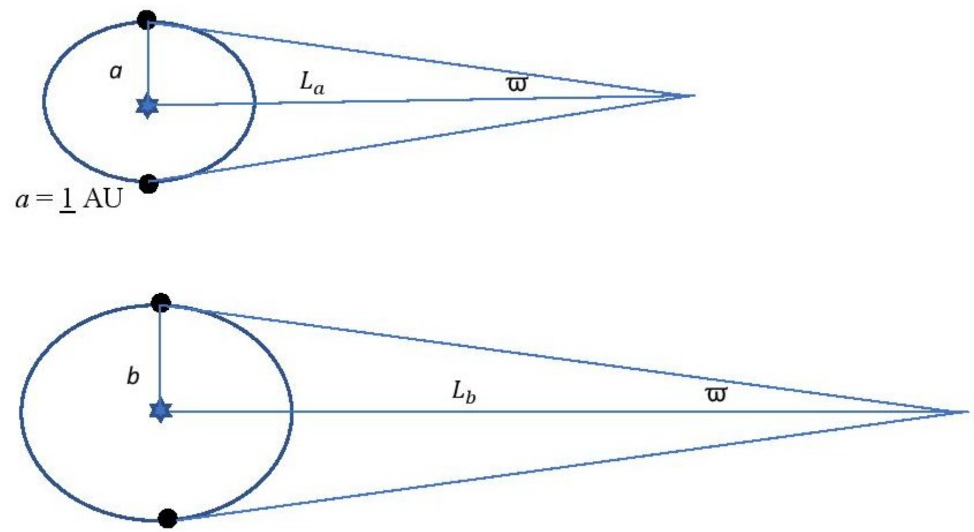
- Introduction

Measuring the distance of an object by trigonometric parallax requires measuring shift in position (change in angle) of less than 1 second.



- Upgrading

The measurement limitation depends on the distance between Sun and Earth.



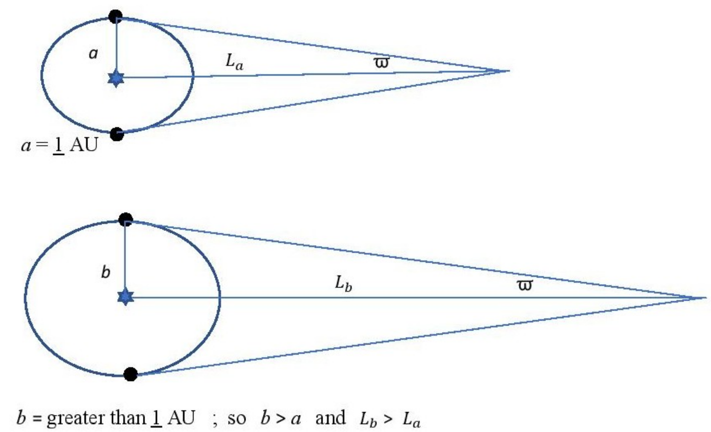
b = greater than 1 AU ; so $b > a$ and $L_b > L_a$

In this figure, if we had an imaginary ability to change the radius of Earth's orbit around the sun, we could extend the limitation.

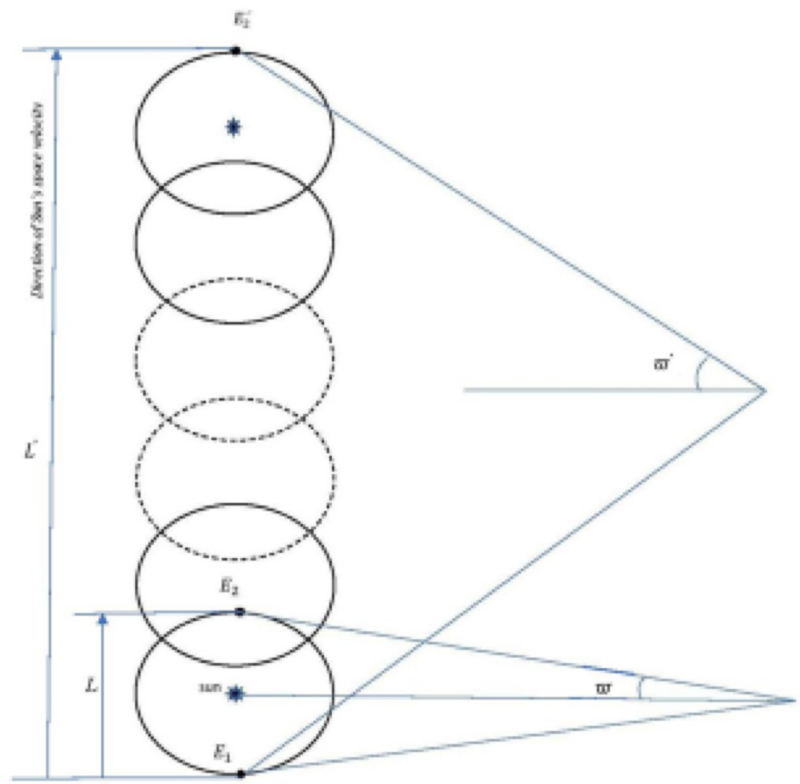
- Can we extend our ability for measuring the distance by this method?
  The answer is Yes.

- If we assume that the Sun is not resident and has a non-zero space velocity around $\underline{20}\ Km\ s^{-1}$ relative the average
  of nearby stars, it means the solar system moves about 4 AU per year, so after twenty years the base line will be forty times larger.
  $2b = 20\times 2\ (\text{AU}) = 40\ \text{AU}\ ,\ 2a = 2\ \text{AU}\ \rightarrow\ 2b = 20\times 2a$

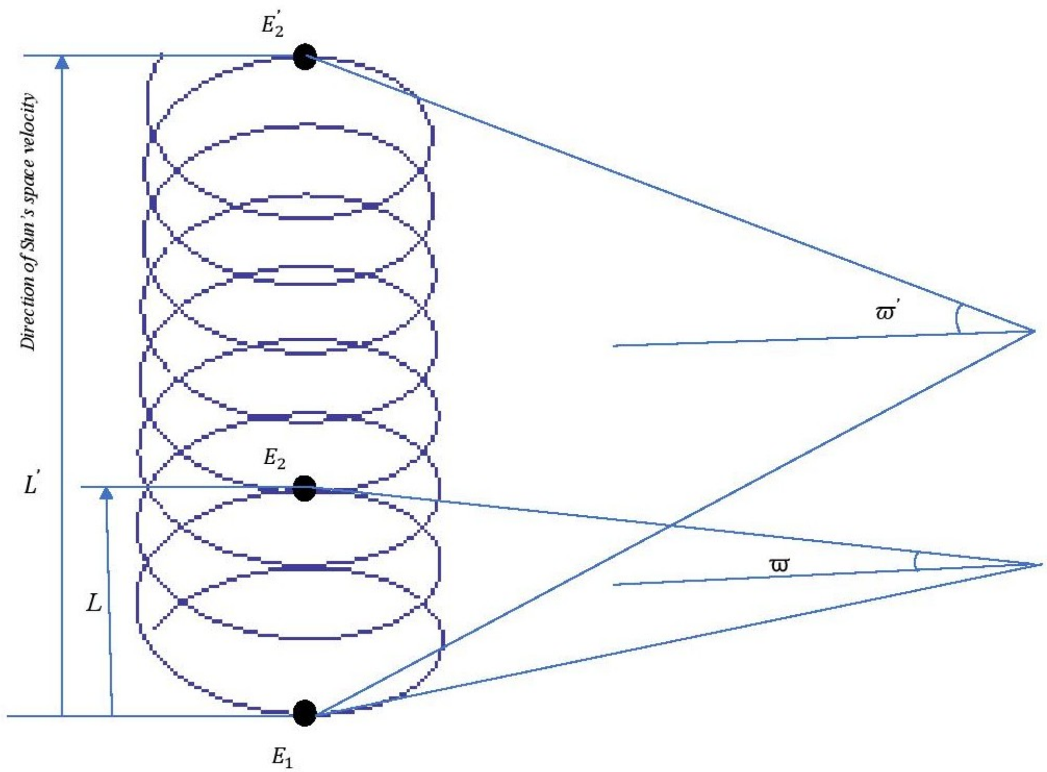
- Again the figure shows two different modes, that ϖ for both is the same for both.
  It means what distance we can measure by a telescope is more in second mode



- The distance $(E_2 - E_1)$ changes to $(E_2^{'} - E_1)$

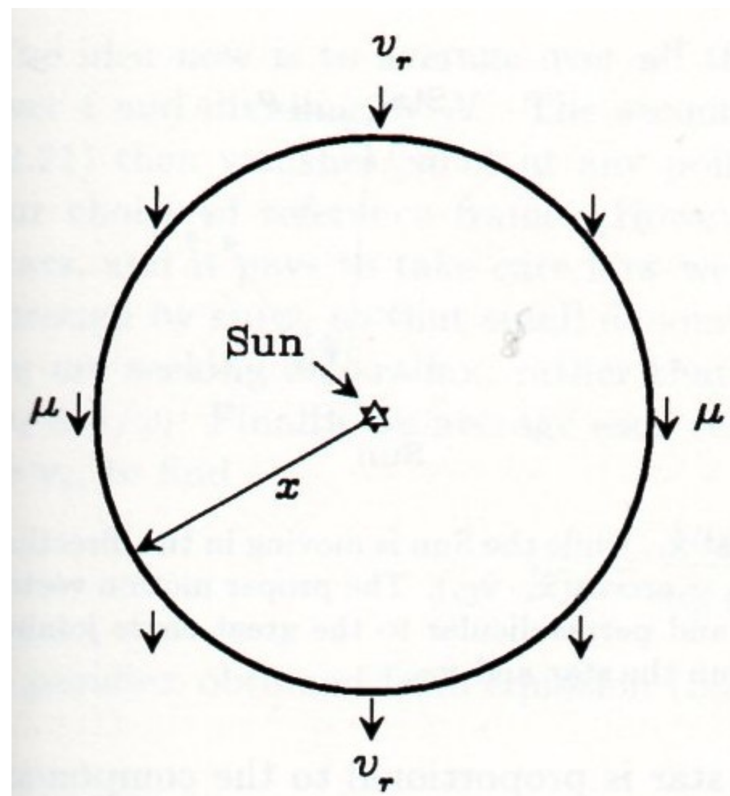


- Actually the path of Earth motion is spiral.



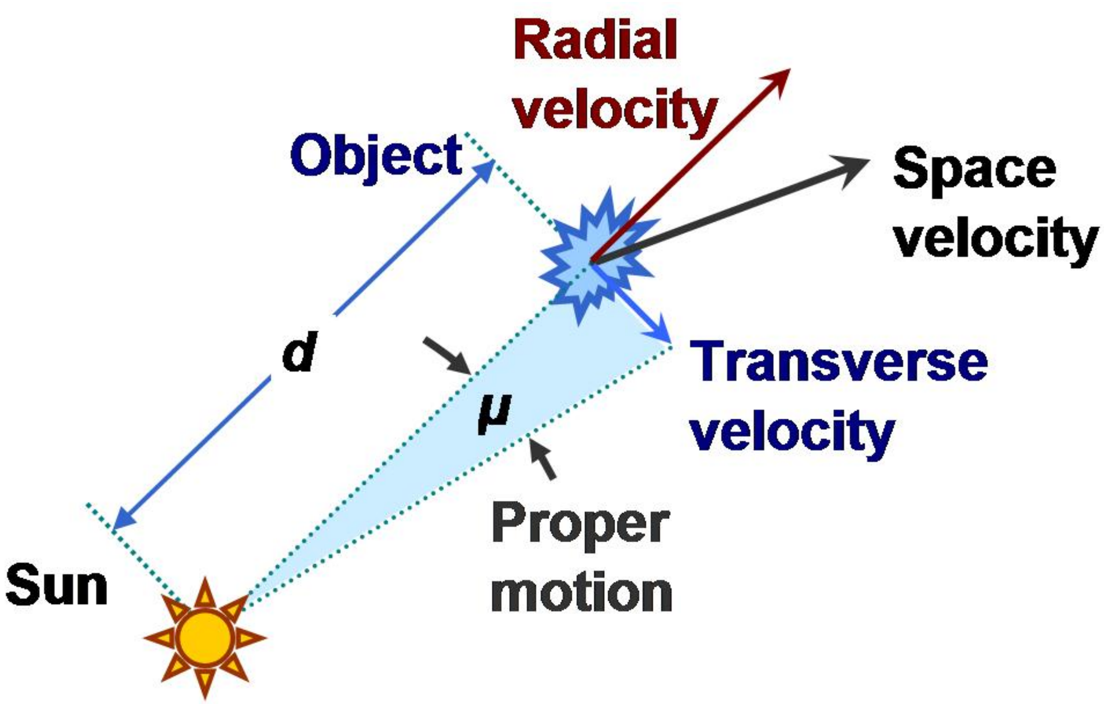
Employing this method, larger baseline, leads us the method “Secular Parallax”.

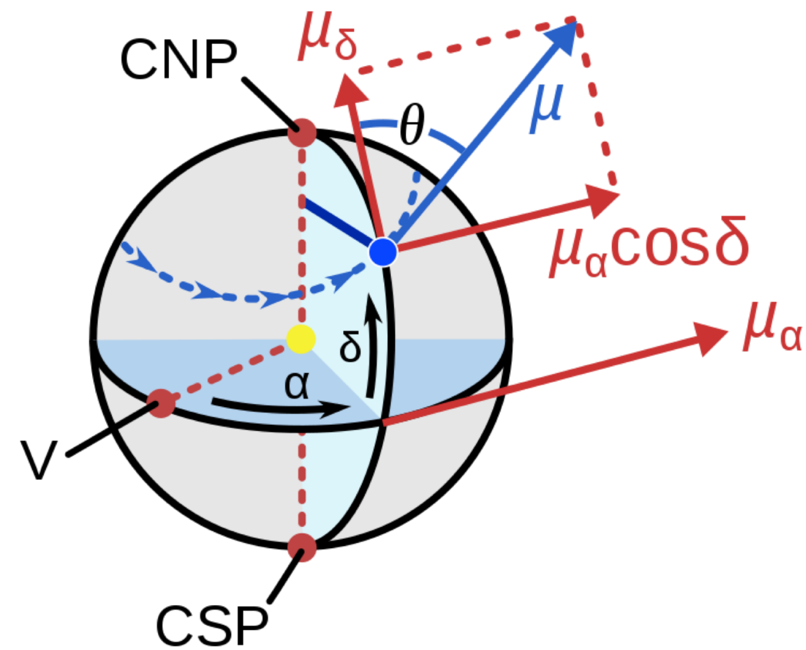
- Secular Parallax
- Suppose the stars that lie within a spherical shell, centered on the Sun with radius *x*, and we choose those stars that have almost the same luminosity and similar apparent brightnesses.
- In this figure Sun moves from bottom to the top so for an observer inside the solar system it seems the circle moves from top to the bottom.



- You know these from previous presentations, only I want to remind that every velocity vector in space has two components (radial velocity and proper motion), while proper motion (μ) is a vector with two components:

$\mu_{\perp} = \mu \cos\theta$; $\mu_{||} = \mu \sin\theta = \mu_{\alpha} \cos\delta$

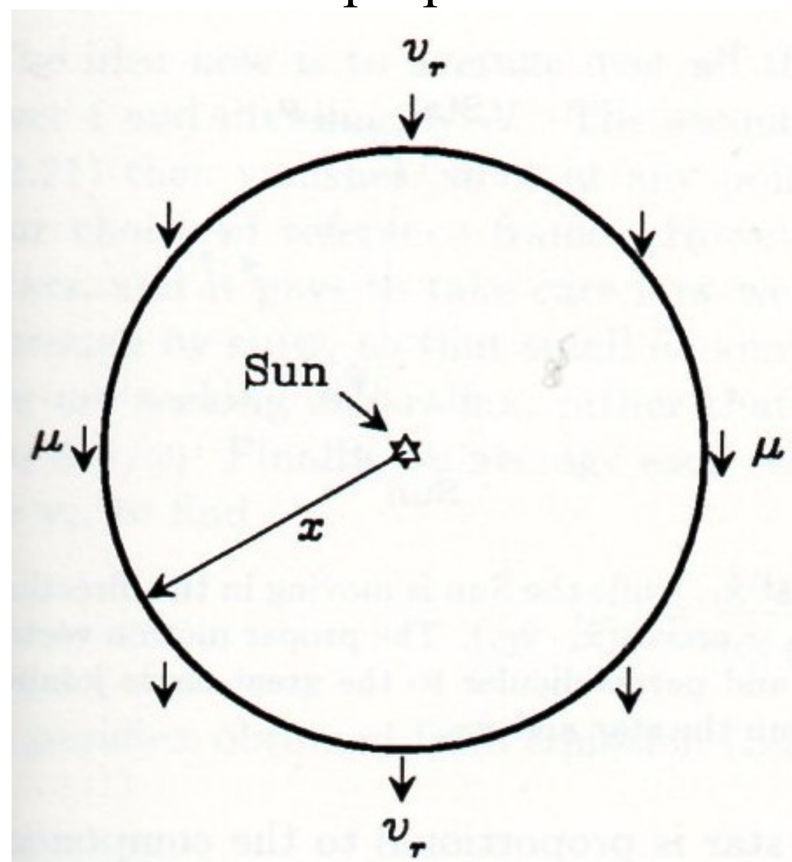




- Now we return to the figure in page 7. Two points at the top and bottom have only radial velocity so we can find the velocity of Sun. Two point at right and left have only proper motion (only $\mu_{\perp}$) so we can find the proper motion and the radius of the sphere.

$$x = \frac{v}{\mu_{\perp}}$$

$\mu_{\perp}$

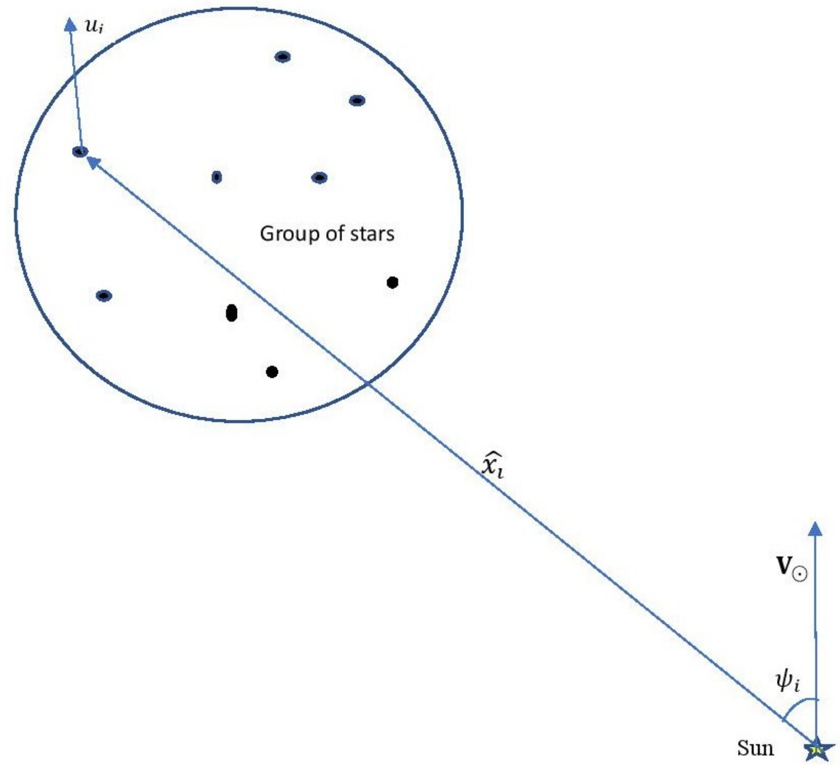


- We have assumed some point on the line of circle, but let assume every point is a group of points. Our investigation will be focused on a frame that is a set of stars which the mean velocity of the stars is zero, $\sum \boldsymbol{V_i} = 0$.

$\boldsymbol{V_i}$ is the vector velocity of i^{th} star

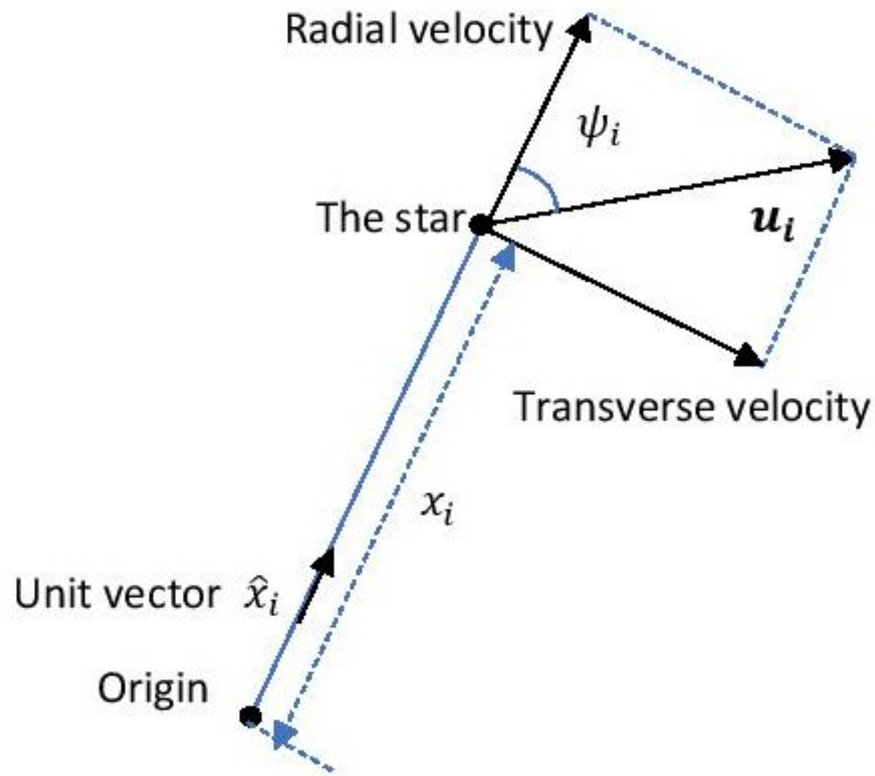
$\boldsymbol{u_i} = \boldsymbol{V_i} - \mathbf{V}_\odot$

$\boldsymbol{u_i}$ is the heliocentric velocity of the i^{th} star



- In principle, the radial velocities of the stars determine $\mathbf{V}_{\odot}$ but usually instead of this procedure, measuring a large number of radial velocities a value for $\mathbf{V}_{\odot}$ is adopted.

- Now we have some algebra
  Transvers velocity = $|\boldsymbol{u_i} \times \hat{x}_i|$ =
  = $|\boldsymbol{u_i}|.\,|\hat{x}_i|.\,\sin\psi_i$



- Continuing Algebra

Proper motion ($\boldsymbol{\mu_i}$) is a vector in direction of transverse velocity

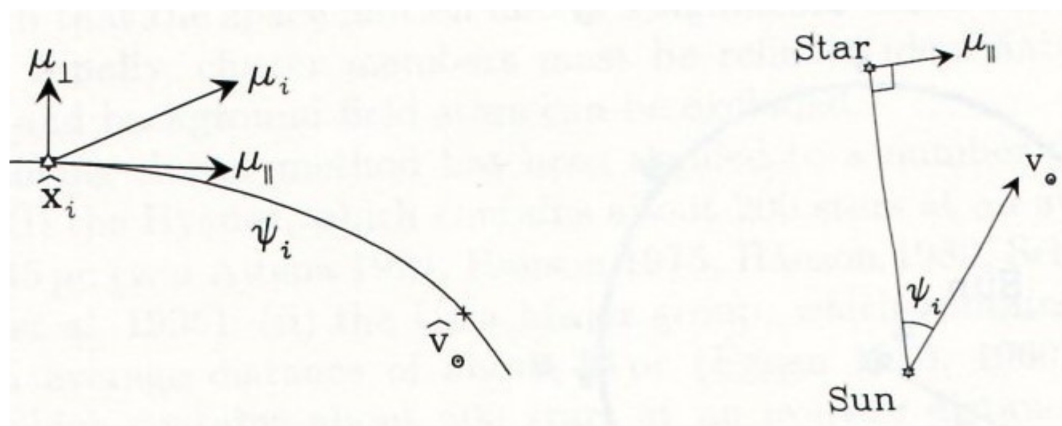
Transverse velocity = $x_i \,.\, \boldsymbol{\mu_i} = |\boldsymbol{u_i} \times \hat{x}_i|$

Considering direction (the vector)

$$x_i \times \boldsymbol{\mu_i} = (\boldsymbol{u_i} \times \hat{x}_i) \times \hat{x}_i \quad \rightarrow \quad \boldsymbol{\mu_i} = \frac{(\boldsymbol{u_i} \times \hat{x}_i) \times \hat{x}_i}{x_i}$$

Substituting $(\boldsymbol{V_i} - \mathbf{V}_\odot)$ instead of $\boldsymbol{u_i}$: $\boldsymbol{\mu_i} = \frac{((\boldsymbol{V_i} - \mathbf{V}_\odot) \times \hat{x}_i) \times \hat{x}_i}{x_i}$

Now we want to find the component $\mu_{\parallel_i}$ that is perpendicular to $\mu_{\perp_i}$



- Finding $\mu_{\|_i}$, the direction is $\mathbf{V}_\odot$ so $\mu_{\|_i} = \boldsymbol{u_i} \,.\, \mathbf{V}_\odot = \frac{\boldsymbol{\mu_i}}{\sin\psi_i} .\, \mathbf{V}_\odot$

$$\mu_{\|_i} = \frac{(((\boldsymbol{v}_i - \mathbf{v}_\odot)\times\hat{x}_i)\times\hat{x}_i)}{x_i \sin\psi_i} \qquad \psi_i = \arccos(\hat{\mathbf{x}}_i \cdot \hat{\mathbf{v}}_\odot)$$

After some calculations: $x_i = \boldsymbol{v}_\odot \frac{\sin^2\psi_i}{\mu_{\|_i}\sin\psi_i} - \frac{(\hat{x}_i\times\hat{\mathbf{V}}_\odot).(\hat{x}_i\times\mathbf{V}_i)}{\mu_{\|_i}\sin\psi_i}$

Our calculation is based on average velocity. With the initial assumption the average of $\mathbf{V}_i$ goes to zero and then second term vanishes.

$$x_i = \boldsymbol{v}_\odot \frac{\sin^2\psi_i}{\mu_{\|_i}\sin\psi_i}$$

- Finally with regard the average of each term and $\varpi_i = \frac{1}{x_i}$

$\langle \varpi \rangle = \frac{\langle \mu_{\parallel i} \sin \psi_i \rangle}{v_{\odot} \langle \sin^2 \psi_i \rangle}$ ψ_i is the angle between $\mathbf{V}_{\odot}$ and $\boldsymbol{X}_i$

Obtaining a parallax by this formula is Secular Parallax method.

Statistical parallax

In this method we use $\mu_{\perp}$ instead of $\mu_{\|}$.

$\boldsymbol{\mu}_{\boldsymbol{r}_{\boldsymbol{i}}} = \hat{x}_i \, . \, (\boldsymbol{V}_{\boldsymbol{i}} - \mathbf{V}_{\odot}) = \hat{x}_i \, . \, \boldsymbol{V}_{\boldsymbol{i}} - \boldsymbol{v}_{\odot} \cos\psi_i$

Considering the component $\mu_{\perp}$ and this assumption that the mean magnitude of any component of $\boldsymbol{V}_{\boldsymbol{i}}$, is the same, then we have

$\langle |\hat{x}_i \, . \, \boldsymbol{V}_{\boldsymbol{i}}| \rangle = \left\langle \left| x_i \, . \, \mu_{\perp_r} \right| \right\rangle$

Therefore with $(\overline{\varpi} = \frac{1}{\bar{x}})$ we have

- $\overline{\varpi} = \frac{\langle \mu_{\perp_i} \rangle}{\langle v_{r_i} - v_\odot \cos\psi_i \rangle}$

- A parallax obtained from this equation is called a statistical parallax.

- If $\langle |\hat{x}\ .\ \boldsymbol{V}| \rangle > \boldsymbol{v}_\odot$, the statistical parallax is probably more reliable.
- If $\boldsymbol{v}_\odot > \langle |\hat{x}\ .\ \boldsymbol{V}| \rangle$, the secular parallax is probably more reliable.

- These two methods have been used to estimate the distance, and also the absolute magnitudes of rare but luminous stars.
- If we compare trigonometric parallax method ($d \approx 20$ pc), and these two methods ($d \approx 500$ pc), there is a considerable improvement for finding the distance of stars from the Earth.