## Error Propagation in Arithmetic Calculations

courtesy of http://www.nuclear.utah.edu/

| Type of Calculation | Example $^{*}$ | Standard Deviation of x |
| :--- | :--- | :--- |
| Addition or Subtraction | $x=p+q-r$ | $s_{x}=\sqrt{s_{p}^{2}+s_{q}^{2}+s_{r}^{2}}$ |
| Multiplication or Division | $x=\frac{p \cdot q}{r}$ | $s_{x}=x \sqrt{\left(\frac{s_{p}}{p}\right)^{2}+\left(\frac{s_{q}}{q}\right)^{2}+\left(\frac{s_{r}}{r}\right)^{2}}$ |
| Exponentiation | $x=p^{y}$ | $s_{x}=y \cdot x \frac{s_{p}}{p}$ |
| Logarithm | $x=\log _{10} p$ | $s_{x}=0.434 \frac{s_{p}}{p}$ |
| Natural Logarithm | $x=\ln p$ | $s_{x}=\frac{s_{p}}{p}$ |
| Antilogarithm | $x=a n t i \log _{10} p$ | $s_{x}=2.303 \cdot x \cdot s_{p}$ |
| Natural Antilogarithm | $x=e^{p}$ | $s_{x}=x \cdot s_{p}$ |

* $\mathrm{p}, \mathrm{q}$, and r are experimental variables whose standard deviations are $\mathrm{s}_{\mathrm{p}}, \mathrm{s}_{\mathrm{q}}$, and $\mathrm{s}_{\mathrm{r}}$, respectively: y is a constant.


## Least Squares Error Analysis

With a set of data points, the error for a linear fit can be found following these equations: ( x and y are the data points and N is the number of data points)

$$
\begin{aligned}
& y=m \cdot x+b: \quad \text { line equation } \\
& \Delta=N \cdot \sum x^{2}-\left(\sum x\right)^{2}: \\
& b=\frac{\sum x^{2} \cdot \sum y-\sum x \cdot \sum x y}{\Delta}: \quad \text { y-intercept } \\
& m=\frac{N \cdot \sum x y-\sum x \cdot \sum y}{\Delta}: \quad \text { slope } \\
& \sigma_{y}=\sqrt{\frac{1}{N-2} \sum_{i=1}^{N}\left(y_{i}-b-m x_{i}\right)^{2}}: \quad \text { standard deviation of the y's } \\
& \sigma_{b}=\sigma_{y} \sqrt{\frac{\sum x^{2}}{\Delta}} \quad: \quad \text { standard deviation of the y-intercept } \\
& \sigma_{m}=\sigma_{y} \sqrt{\frac{N}{\Delta} \quad: \quad \text { standard deviation of the slope }} \\
& y=\left(m \pm \sigma_{m}\right) \cdot x+\left(b \pm \sigma_{b}\right): \quad \text { line equation with error }
\end{aligned}
$$

This system of equations is what Microsoft Excel uses in its LINEST function.

