A Multiple-Star Combined Solution Program - Application to the Population II Binary μ Cas

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Abstract

A multiple-star combined-solution computer program which can simultaneously fit astrometric, visual, and spectroscopic data, and solve for the orbital parameters, parallax, proper motion, and masses has been written and is now publicly available. Some features of the program are the ability to scale the weights at run time, hold selected parameters constant, handle up to five spectroscopic subcomponents for the primary and the secondary each, account for the light travel time across the system, account for apsidal motion, plot the results, and write the residuals in position to a standard file for further analysis. The spectroscopic subcomponent data can be represented by reflex velocities and/or by independent measurements. A companion editing program which can manage the data files is included in the package.

The program has been applied to the Population II binary μ Cas to derive improved masses and an estimate of the primordial helium abundance.

The source code, executables, sample data files, and documentation for OpenVMS and Unix, including Linux, are available at http://www.chara.gsu.edu/~gudehus/binary.html.

1. INTRODUCTION

Several methods, both graphical and analytical, exist for solving for the elements of binary star systems. Descriptions of some of the early ones can be found in Petrie (1962), Smart (1962) and Aitken (1964). For example, for a visual binary one can use the method of Thiele and Innes, where the geometric elements, given as a function of the Thiele-Innes rectangular constants, and the dynamical elements are derived in two stages. For a spectroscopic binary the method of Lehmann-Filhés can be used. When astrometric data is available, the parallax and proper motion can be solved for simultaneously with the Thiele-Innes constants. The above visual binary methods, however, do not directly yield a covariance matrix for all the elements and do not readily lend themselves to handling large numbers of independently weighted data points. The Lehmann-Filhés method does not directly solve for all the elements, but rather includes in its solution the reciprocal of the period, and the amplitude, K, which is a function of several elements. These methods can, on the other hand, serve to provide initial estimates for more rigorous least squares reductions.

Early least squares methods for visual binaries often retained use of the Thiele-Innes constants and/or considered the inverse of the period as an element, e.g. Wielen (1962), or relied on lookup tables for the derivatives when making differential corrections (Irwin 1961). Early least squares solutions for spectroscopic binaries tended to retain K as solution element, or possess biases in the treatment of the equations (see Popper (1974) for a discussion).

More recently, least squares methods by Barlow (Barlow and Scarfe, 1991) and Tokovinin (1992, 1993) have been written which permit combined visual and spectroscopic binary solutions. The purpose of this paper is to present and make publicly available a least squares combined solution computer program which can handle combined astrometric, visual, and spectroscopic multiple star data.

2. IMPLEMENTATION OF NONLINEAR LEAST SQUARES

2.1 Nonlinfit

The method of least squares, first concieved of by Gauss, is well known and follows from the more general maximum likelihood method in the case where the observations are samples from a multi-variate normal distribution.

In this case the least squares estimate is the same as the maximum likelihood estimate. Even when the distribution is not normal, the Gauss-Markov theorem says that that the least squares method still gives the most efficient unbiased linear estimators. When the condition equations are nonlinear, one could in principle search in nonlinear space for a minimum variance. This is the approach of the CERN least squares fitting package MINUIT. Tests conducted by Mr. Slava Sharkin and the author many years ago on a version MINUIT modified to function as a subroutine showed however that convergence with this package is rather slow and the accuracy of the solution is not as great as that of the subroutine Nonlinfit (Gudehus 1987).

Nonlinfit achieves a nonlinear least squares solution by expanding the the condition equations in a first order Taylor series about a preliminary solution, and iterating until an acceptable convergence criterion is achieved. The function and the derivatives must be hard coded in subroutines for each application. While it is possible to calculate the derivatives numerically, only analytical formulations were used for the multiple star combined solution application. Nonlinfit can operate interactively or automatically and through the use of dynamic memory, the number of observations, parameters, and dependent and independent dimensions is limited only by the computer main memory.

If Nonlinfit is asked to solve a linear least squares problem, e.g. a polynomial, the solution is obtained after the first iteration. For nonlinear problems, not only will several iterations be required, but multiple minima will in general be present. In Nonlinfit the decision whether to continue iterating can be made manually (interactively) or automatically. In the latter case, arrival at a limiting value of the ratio of the change of the parameters to their value plus mean error (fractional tolerance) will signal convergence. In well behaved situations the parameters will monotonically approach their final values, but in general it may be necessary to relax the solution to prevent excessive bouncing around. In the automatic iteration mode, the fractional parameter change from one iteration to the next (fractional increment) is automatically adjusted to achieve convergence. The adjustment can be done so as to optimize for speed, or to optimize for searching among nearby minima. In the latter case, the best fit parameters are retained at each stage, but the solution is allowed to temporarily wander to parameters with larger mean errors, in a style of simulated annealing (Metropilis, 1953), cf Pourbaix (1994).

2.2 Application to Astrometric Data

Astrometric data, obtained from measuring engines or by other means, is read by the program as X and Y coordinates in some chosen angular unit, along with time measured in years. Data, with parallax and proper motion included, or not included, can be handled by the program. The program uses the data format and header keywords to decide what form the data is in. For a single star, up to fifteen parameters (period, P; time of periastron passage, T; eccentricity, e; angle of line of nodes, Ω; inclination, i; angle from node to periastron, ω; semimajor axis, a; parallax, π; X and Y center, proper motion, and acceleration; and apsidal motion) can be solved for. For a pair of stars, R, the ratio of semimajor axes (total to primary) becomes an additional parameter.

2.3 Application to Visual Data

Visual data is usually in the form of position angle in degree and separation of the secondary relative to the primary in some chosen angular unit, and time in years, but the program will also accept X and Y coordinates. Astrometric data with parallax and proper motion removed is not completely equivalent to visual data for the primary in the form of X and Y coordinates since the former is absoute position data and the latter, relative. Data for one star, e.g. speckle data, can be modeled with up to eight parameters (P, T, e, Ω, i, ω, a, and apsidal motion. For a pair of stars, R is added to the list of parameters.

2.4 Application to Spectroscopic Data

Spectroscopic data can be in the form of velocity in any chosen unit, and time in either Julian Days or Besselian years. A soution for a single-lined binary is modeled by up to seven parameters (P, T, e, ω, system radial velocity, V; the product $a\sin i$, with a in units of AU; and apsidal motion). The program will also calculate K, though this is not an element and is never used as a parameter in the solution. For a double-lined binary, R is added to the list of parameters.

The program will make allowance for the light travel time across the orbit if requested, and also allows for the possibility of subcomponents whose data may be present either as reflex velocities in the primary or secondary data, or in separate files. Up to five subcomponents per binary component are allowed, for a total of twelve stars. Up to three ranges of phase may be excluded from the solution to test for data compromised by line blending.

2.5 Combined Solution Considerations

Because in a combined solution various independent sets of data are being used for one solution, it is important to establish a uniform set of definitions for the various elements. Thus, a is taken to be that of the primary when possible, and is complemented by R when available. The angle ω is also taken to be that of the primary, even when only data for the secondary exists.

Combined astrometric or visual plus spectroscopic data allow one to solve for the parallax. The program automatically determines when this condition applies from the header and keyword items in the data files, and from the user choices as to whether a particular file pertains to a primary or secondary. This task, the semimajor axis assignment, the calculation of dimensions and offsets, and other logical states are carried out in a subroutine dedicated to that purpose.

For astrometric data, the equator for the X and Y positions is assumed to be that of the the star's RA and Dec, given in the header. On the other hand, for visual (or speckle) data, the equator is assumed to be that of the epoch of observation. Astrometric data has two characteristics which must be taken into account when solving for the parallax and semimajor axis in a combined solution. Astrometric parallax is affected by the the motions of the background standard stars, but the parallax derived from combined visual and spectroscopic data is not. Thus the ratio of the apparent to true parallax must be provided to the program. An astrometrically derived semimajor axis is a photocentric value, whereas the value from combined visual and spectroscopic data is a true value. Again the program must be provided with the ratio of the photocentric to true semimajor axis.

In a least squares application with more than one data dimension, the possible presence of different data units with different standard deviations, requires that one minimize the sum of the squares of the residuals in an unbiased way. Therefore for a total of N_k observations i of dimension k, and a total number of dimensions equal to N_D, the sum of the squares of the observed minus calculated points,

$$\sum_{i}^{N_k}\sum_{k}^{N_D}(\mathcal{O}_{ik}-\mathcal{C}_{ik})^2 w_{ik}$$

should be minimized. Here, w_{ik} is the weight of observation i of dimension k, which is equal to the inverse square of the standard deviation, σ_{ik}. The standard deviation for a particular dimension could be determined from a solution of the binary system with only its matching kind of data (astrometric, visual, or spectroscopic) if enough points are available, or from estimates based on similar data. Note that since normal points are never used, there are never any additional dimensionless weighting factors in the above equation.

In the absence of any systematic effects, and with properly weighted data, the reduced chi squared, χ_r^2, should be approximately unity for each dimension, both for solutions of one particular kind, and for a combined solution. In the event that χ_r^2 is nonunity, one can investigate possible improper weights, and/or systematic effects by making use of program features that allow scaling of the weights during run time, and exclusion of data points which deviate by a given factor beyond the mean error. In addition, a companion editing program, "editdata", permits one to permanently adjust values in the data files. Individual data points can be excluded by setting their sigmas to the key value 9999.99999.

3. APPLICATION TO μ Cas

3.1 Scientific Importance

μ Cas (HD 6582), a nearby high velocity population II star, is interesting because a determination of the masses, luminosities, and metal abundance of its components allows its helium abundance to be determined. Since the age of the star predates that of the disk of the Galaxy, the abundance is nearly primordial and is therefore of cosmological interest (Dennis 1965).

3.2 Determination of Masses

In principle the masses of a binary could be found by combining a single measurement of the separation of the components with a knowledge of the astrometric orbit. For μ Cas, however, it was soon discovered that the masses depended on whether the orbit of Lippincott (1981) or of Russell & Gatewood (1984) was used (McCarthy 1984). Haywood et al. (1992) pointed out that the two astrometric orbits differed mainly in their values of ω, and proposed solving for ω plus the semimajor axis of the relative orbit while holding all the other parameters constant, using the combined astrometric and relative orbit data. This approach yielded, for the first time, masses that were much more consistent between the two sets of data.

Since the Haywood et al. study, another astrometric orbit has been published by Heintz and Cantor (1994), and additional relative orbit measurments have become available by McCarthy et al. (1993) and Drummond et al. (1995). In addition, spectroscopic measurements by Jasniewicz & Mayor (1988) and Duquennoy et al. (1991), which had not been considered in the mass determinations, are available. With the additional new data and the complete astrometric data sets kindly made available by Sara Lee Lippincott, Wulff Heintz, and Drs. Russell & Gatewood, it seemed promising to try a combined solution with the new software.

For Lippincott's data, the ratio of apparent to true parallax is calculated from her published results for μ Cas to be 0.9841; for the Heintz and Cantor data this value is 0.9845; and for the Russell & Gatewood data the value is 0.978. The ratio of photocentric to true semimajor axis, based on the temperatures and radii of components A and B from Haywood et al., is calculated to be 0.998. With these values and the average weights scaled so as to yield $\chi_r = 1$ for each dimension, the orbital elements and other parameters shown in Table 1 were obtained in combined solutions for each data set. Also shown are the mean errors for an observation of average weight, $MEOAW$. As Haywood et al. found earlier with their restricted solution, the masses for the Lippincott and Russell & Gatewood data agree very well. The errors here however are somehat smaller. The mass agreement for the primary is slightly fortuitous because the Russell & Gatewood values of primary a, and π, are smaller and larger, respectively, than those of Lippincott, which together with a value of R that is larger than that of Lippincott, give a canceling effect for the mass variation. For the Heintz & Cantor data, a slightly larger π and smaller R combine to give a somewhat lower primary mass than that from Lippincott.

Russell & Gatewood's astrometric $MEOAW$s are substantially larger than those of the other data sets but this is compensated for by a larger number of observations. Of the 370 observations in their data, seven were excluded because of much larger deviations from the calculated orbit. Heintz and Cantor fixed the proper motion acceleration in their solution at the true perspective values. A combined solution with this restriction increased the

$MEOAW$ in the X direction by 20% and decreased the primary mass, $\mathcal{M}_{\mathrm{A}}$, to 0.68 $\mathcal{M}_{\odot}$.

The last line in Table 1 shows the helium abundance calculated from the interpolation formula and the bolometric magnitude from Haywood et al.'s paper. The value of metal abundance, $Z = 0.0021$, is an average of the results of Perrin et al. (1977) and Tomkin & Lambert (1980). The values of Y from the Lippincott and Russell & Gatewood data sets are not greatly different from what one might expect from current cosmogical predictions. The errors in Y are computed solely from the uncertanties in mass and parallax and thus do not include errors originating from the bolometric correction, the stellar models of VandenBerg (1985) and VandenBerg & Bell (1985), or the interpolation formula.

Finally, Figure 1 shows a portion of Russell & Gatewood's data and the visual data in the time interval 1974 to 1995 in absolute coordinates. Figure 2 shows Lippincott's data with parallax and proper motion removed, and the visual data in absolute coordinates. Figure 3 shows the spectroscopic data transposed to cover one cycle.

4. SUMMARY

A combined solution approach to binary star research can be beneficial in two ways: 1) by making maximal use of the available data to yield improved orbital parameters and mass estimates and 2) by revealing inconsistencies between data sets through the use of the program diagnostic values of $MEOAW$ and χ_r. The program, which has been developed to run on both OpenVMS and Unix platforms comes with full plotting capability. The instructions, source code, executables, and sample data are available at the author's web site at http://www.chara.gsu.edu/~gudehus/binary.html. A companion binary star editing program is included as well.

New combined solution estimates of the masses of the components of μ Cas from three different data sets show only small differences from each other. When averaged together, they are

$$\mathcal{M}_{\mathrm{A}} = 0.748 \pm 0.027\mathcal{M}_{\odot}$$

$$\mathcal{M}_{\mathrm{B}} = 0.1698 \pm 0.0051\mathcal{M}_{\odot}.$$

Dedicated to Daniel Popper

TABLE 1

μ Cas COMBINED SOLUTIONS

Parameter	Data Set I	Data Set II	Data Set III
P (years)	21.480±0.042	21.467±0.046	21.858±0.048
T (years)	1975.727±0.064	1975.708±0.068	1976.076±0.097
e	0.5904±0.0098	0.596±0.010	0.514±0.013
$\Omega(^\circ)$	47.27±0.68	46.47±0.67	47.63±0.85
$i(^\circ)$	−109.69±0.63	-110.45 ± 0.67	-109.49 ± 0.89
$\omega(^\circ)$	334.3±1.6	333.9±1.7	337.35±2.2
$a('')$	0.1851±0.0021	0.1868±0.0021	0.1823±0.0027
R	5.36±0.12	5.29±0.13	5.66±0.15
$\pi('')$	0.1311±0.0016	0.1332±0.0019	0.1353±0.0024
$\mu_\alpha(''/\mathrm{y})$	3.42486±0.00079	3.42236±0.00049	3.4721±0.0012
$\dot{\mu}_\alpha(''/\mathrm{y}^2)$	0.000019±0.000020	−0.000068±0.000015	0.000474±0.000030
$\mu_\delta(''/\mathrm{y})$	-1.58789 ± 0.00088	−1.58804±0.00050	−1.6080±0.0011
$\dot{\mu}_\delta(''/\mathrm{y}^2)$	-0.000067 ± 0.000023	−0.000057±0.000014	−0.000322±0.000026
$V_r(\mathrm{km\ s^{-1}})$	−98.236±0.092	−98.249±0.094	−98.065±0.087
N	257	293	405
$MEOAW_{\mathrm{A},X}('')$	0.01516 ± 0.00074	0.01847 ± 0.00083	0.0313 ± 0.0012
$MEOAW_{\mathrm{A},Y}('')$	0.01733 ± 0.00084	0.01997 ± 0.00090	0.0289 ± 0.0011
$MEOAW_{\mathrm{B},X}('')$	0.039 ± 0.010	0.040 ± 0.010	0.042 ± 0.011
$MEOAW_{\mathrm{B},Y}('')$	0.051 ± 0.013	0.050 ± 0.012	0.0303 ± 0.0076
$MEOAW_{\mathrm{A},V}(\mathrm{km\ s^{-1}})$	0.498 ± 0.067	0.511 ± 0.068	0.450 ± 0.060
$\mathcal{M}_\mathrm{A}(\mathcal{M}_\odot)$	0.764±0.059	0.719±0.059	0.767±0.069
$\mathcal{M}_\mathrm{B}(\mathcal{M}_\odot)$	0.175±0.010	0.168±0.010	0.165±0.012
$Y_{\mathrm{A},Z=0.0021,t=8}$	0.236±0.005	0.278±0.004	0.226±0.006
$Y_{\mathrm{A},Z=0.0021,t=10}$	0.222±0.005	0.265±0.004	0.211±0.006
$Y_{\mathrm{A},Z=0.0021,t=12}$	0.208±0.005	0.253±0.005	0.198±0.006

Data Set I = Astrometric: Lippincott (1981); Speckle: Wickes & Dicke (1974); Wickes (1975); Haywood et al. (1992); Karovska et al. (1986); McCarthy et al. (1993); Drummond et al. (1995); Spectroscopic: Jasniewicz & Mayor (1988); Duquennoy et al. (1991)

Data Set II = Same as I except Heintz & Cantor (1994) instead of Lippincott (1981)

Data Set III = Same as I except Russell and Gatewood (1984) instead of Lippincott (1981)

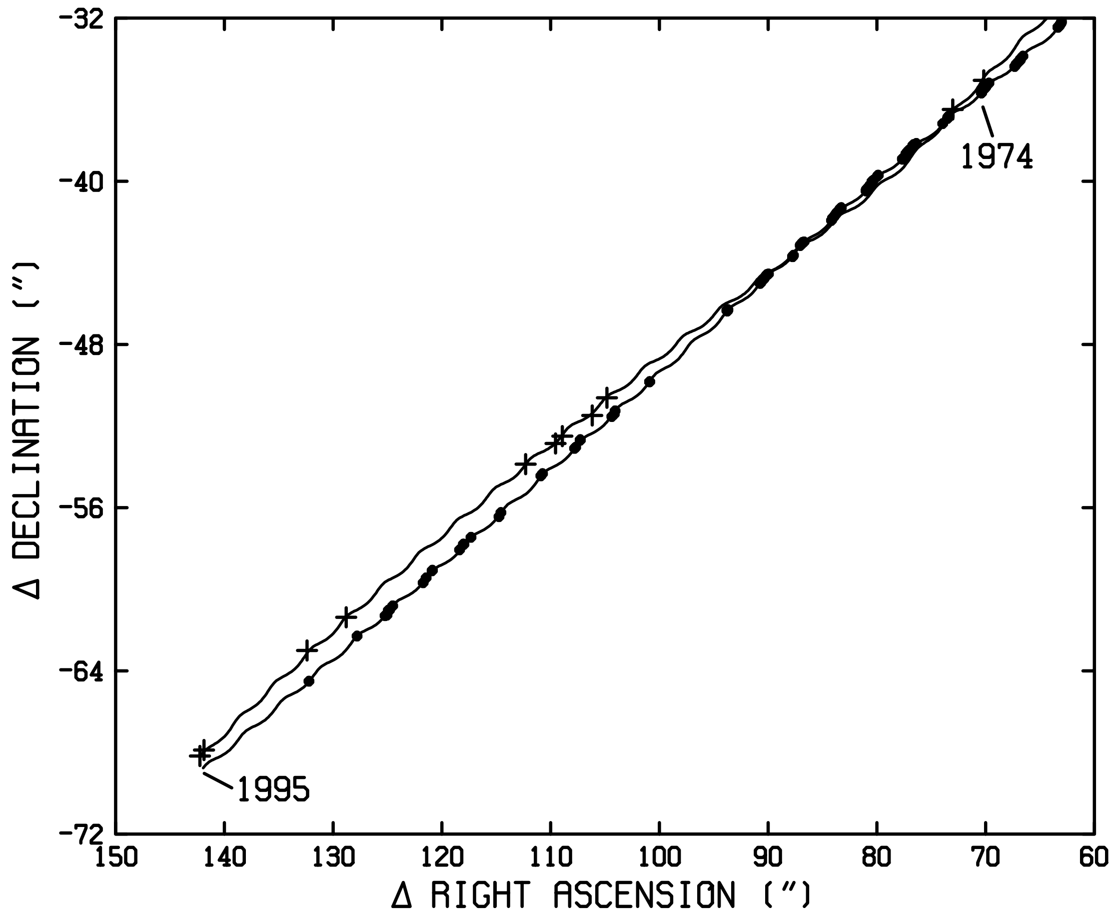

Fig. 1 A portion of the orbit of μ Cas in absolute coordinates for the time period 1974 to 1995. The astrometric data for the primary was provided by Wulff Heintz, and the secondary data are from the various observations noted in Table 1.

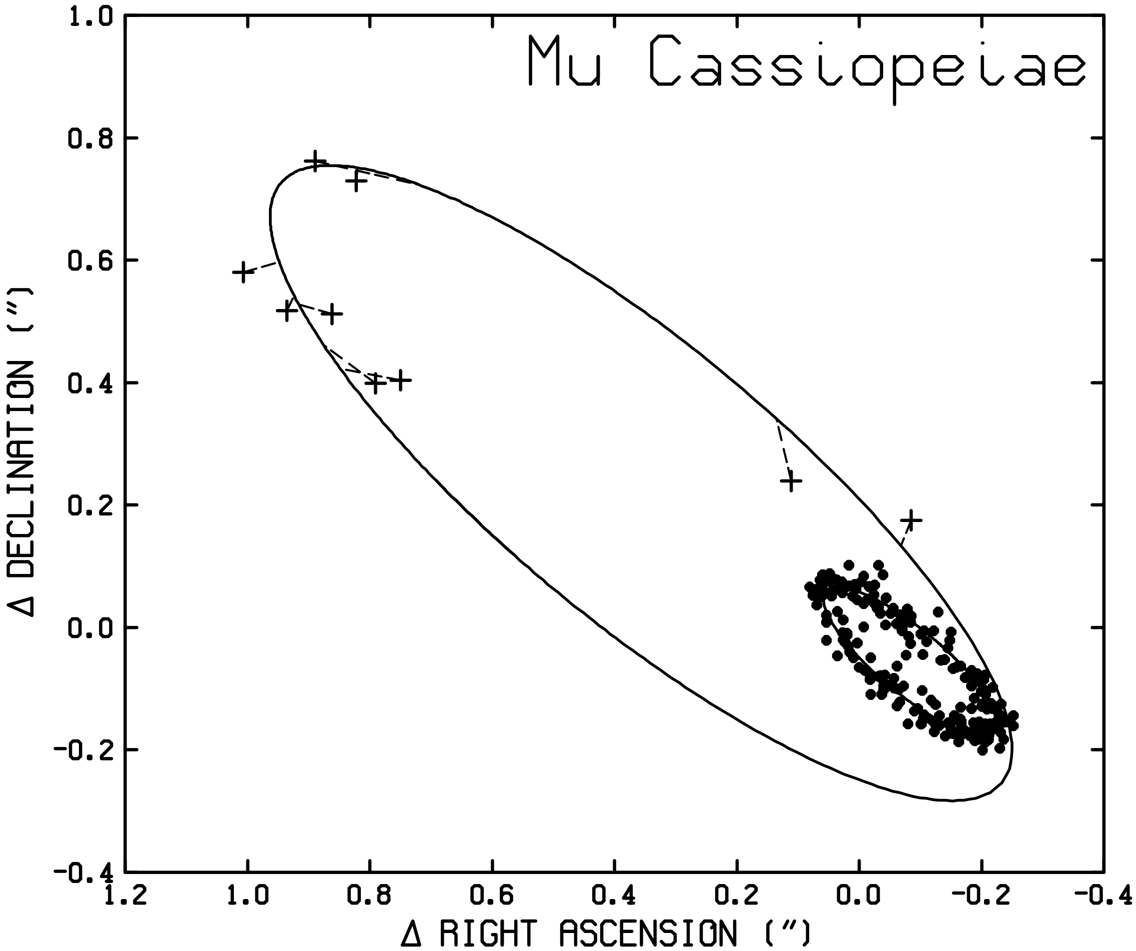

Fig. 2 The orbits of μ Cas A and B in absolute coordinates. The astrometric data for the primary was provided by Sara Lee Lippincott, and the secondary data are from the various observations noted in Table 1.

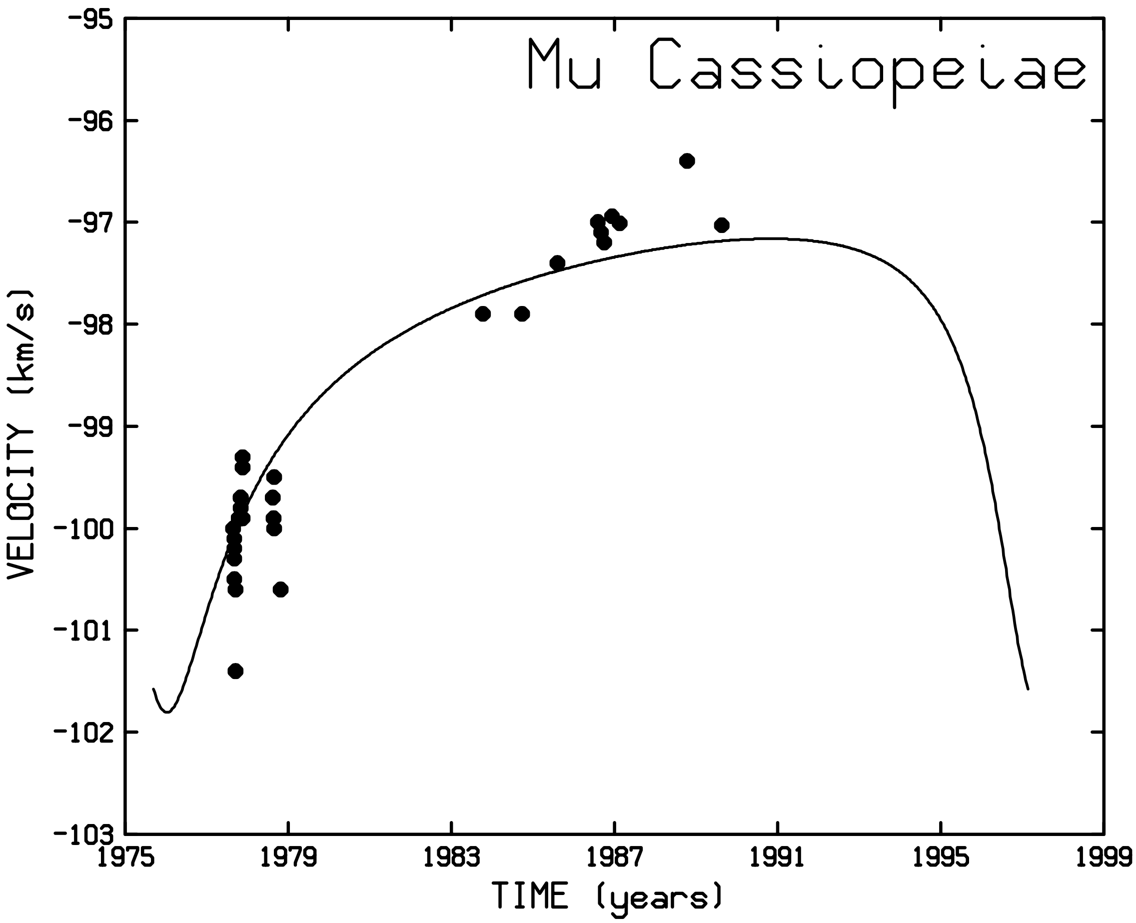

Fig. 3 Radial velocity measurements of Jasniewicz & Mayor (1988) and Duquennoy et al. (1991) of μ Cas A transposed to cover one cycle.

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