Hierarchical Bayesian Models with JAGS/BUGS Applications to Binary Stars & Asteroseismology

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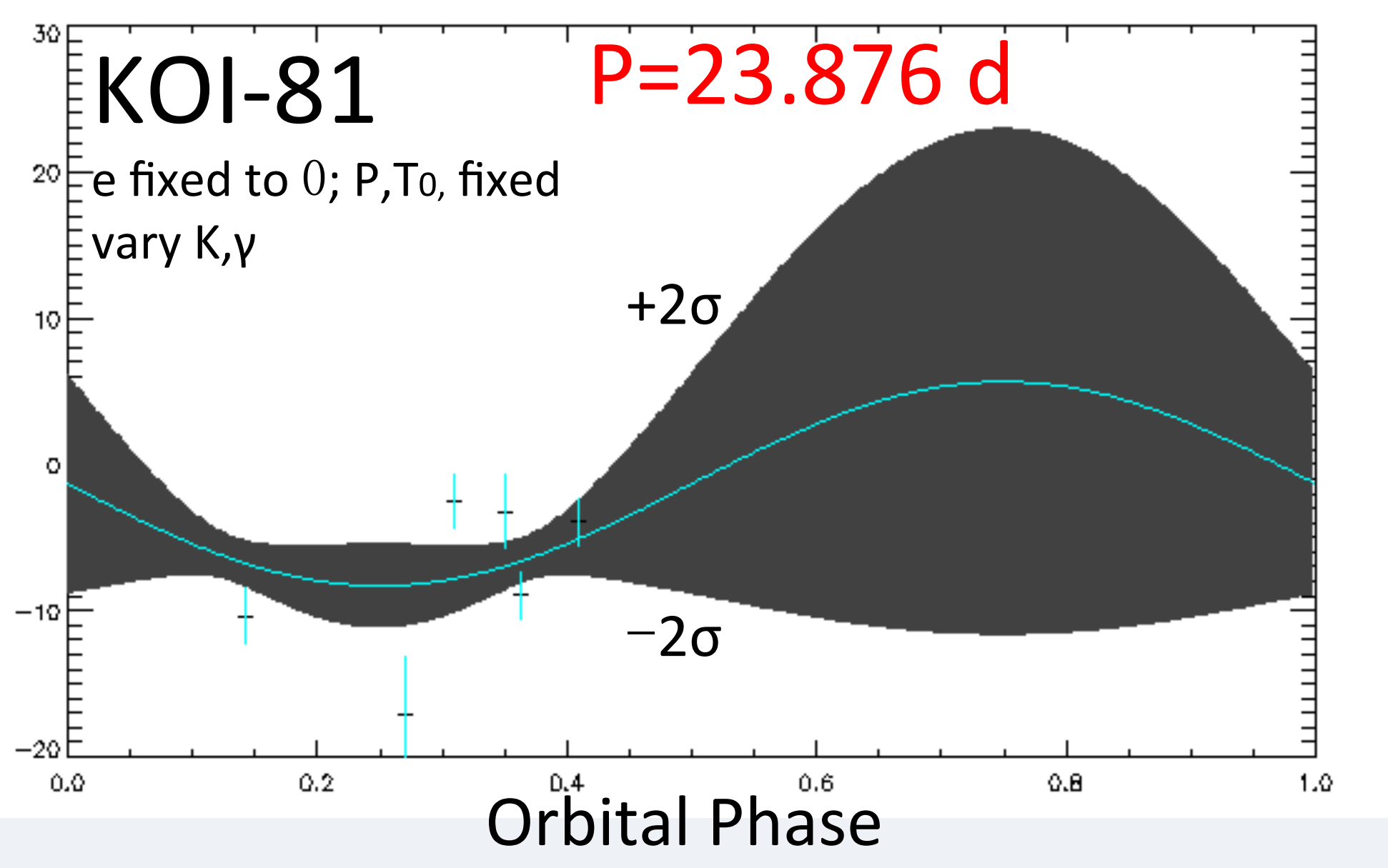
1. *Fitting Radial Velocities*

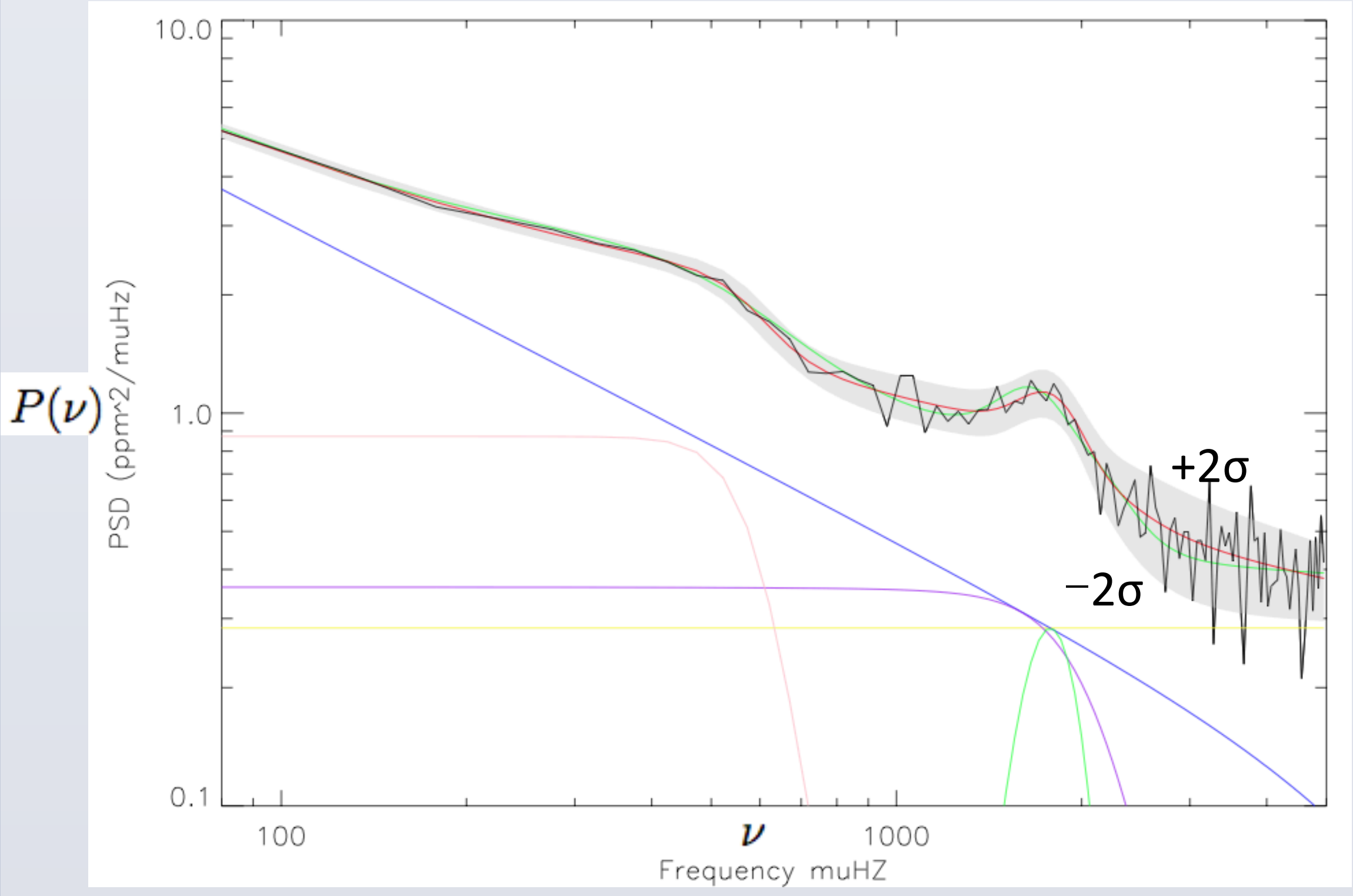


$$V(t) = [K(\cos(\omega + f(t))) + e\cos(\omega)] + \gamma$$

The spectral lines of stars will be blue/red shifted if the stars move towards or away from us. These shifts correspond to the Radial Velocities, which is the most important method to determine stellar masses.

We applied the JAGS sampler to fit the Radial Velocity curve of KOI-81, an eclipsing binary system composed of a B star and a white dwarf companion. For data of bad phase coverage, this tool gives reliable error estimates.

2. *Asteroseismology: Fitting Oscillation Background*



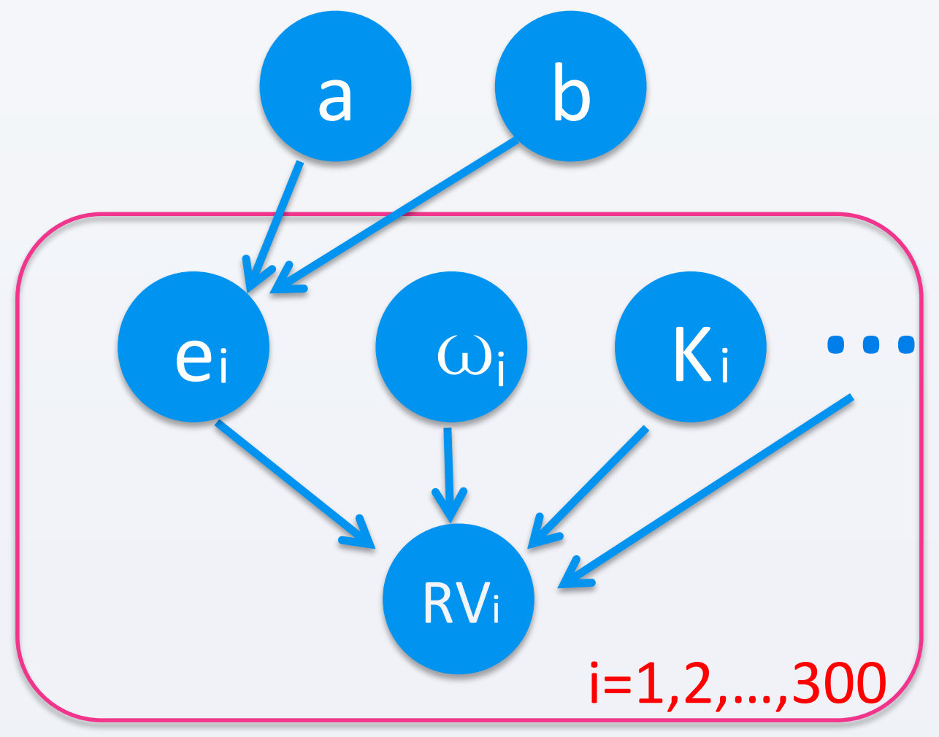
$$P(\nu) = W + sinc^2\left(\frac{\pi\nu}{2\nu_{nq}}\right)\left(a\nu^{-b} + \frac{4\tau_1\sigma_1^2}{1+(2\pi\nu\tau_1)^2} + \frac{4\tau_2\sigma_2^2}{1+(2\pi\nu\tau_2)^2} + H_{osc}exp(-\frac{\nu-\nu_{max}}{2\sigma_{env}^2})\right)$$

Noise Two Lorentzians Power excess

In Asteroseismology, it is often needed to fit the background of the power density spectrum of solar-like oscillators.

The background consists of a noise term, W, two Lorentzian functions, and a power excess due to oscillations. JAGS works well for this problem (12 free parameters).

2. Retrieving the Eccentricity Distribution from Large Surveys



$$RV \sim N(RV|RV_{model}, C_{RV})$$
$$RV_{model} = f(e, \omega, K, \gamma, P, T_{peri})$$
$$C_{RV} = I_{\sigma_{RV}}$$

Correlated noise can be accounted for here using more detailed covariance matrix, e.g. from Gaussian Process or ARMA.

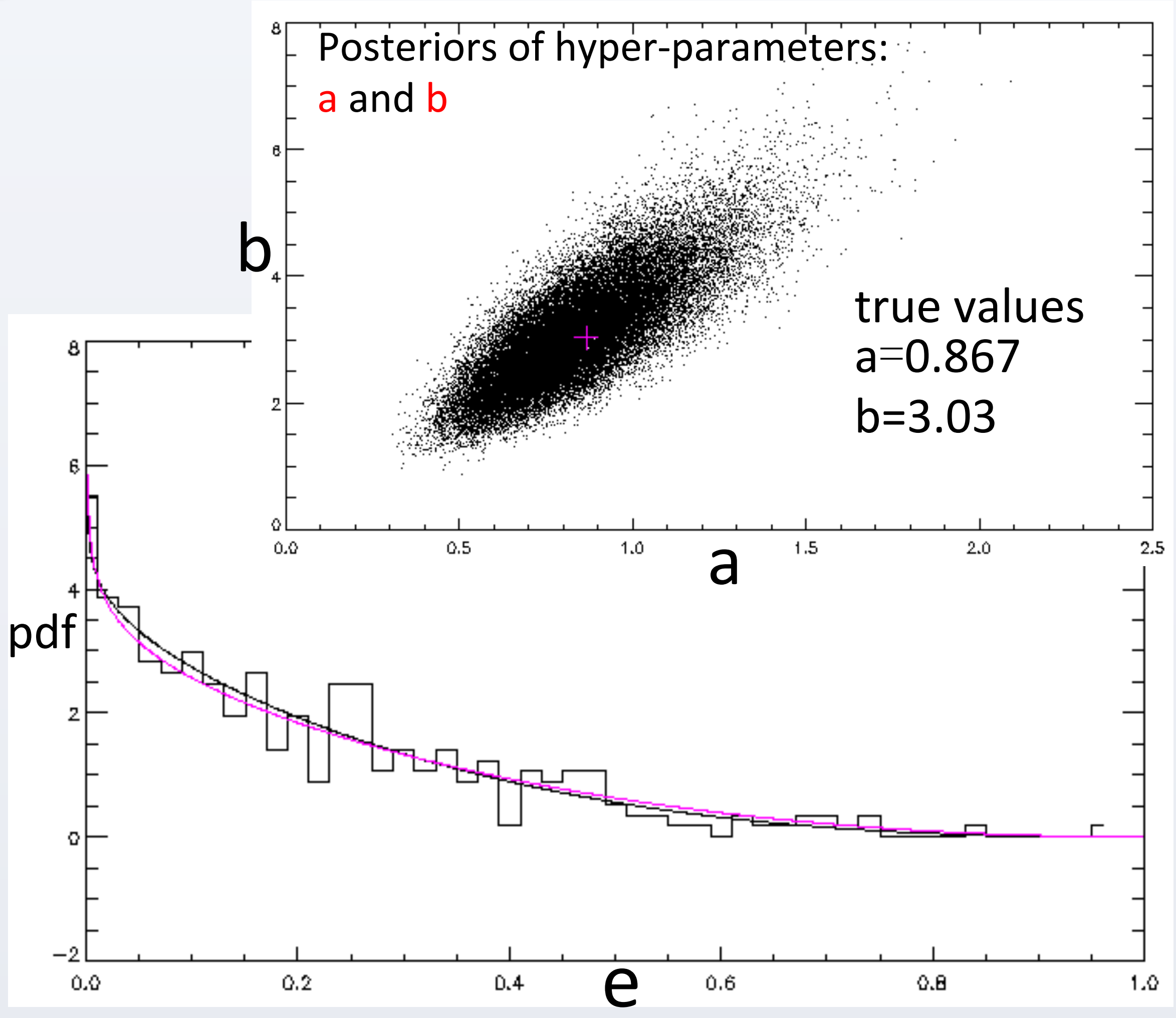
$$e \sim Beta(e|a, b)$$
$$\omega \sim Unif(0, 2\pi)$$
$$K \sim Unif(K_{min}, K_{max})$$
$$\gamma \sim Unif(\gamma_{min}, \gamma_{max})$$
$$P \sim Unif(P_{min}, P_{max})$$
$$T_{peri} \sim Unif(T_{min}, T_{max})$$

prior

$$a \sim Unif(0.1, 8)$$
$$b \sim Unif(0.1, 8)$$

hyper-prior

We generated artificial RVs of 300 planet host stars with eccentricities from an underlying distribution Beta(0.867,3.03) (Kipping 2013, MNRAS, 434, L51). We then fit each RV curve to find orbital parameters. We use a population prior on eccentricity applicable to each system Beta(a,b) and hyper-priors on a and b.



The histogram in the lower panel shows the distributions of all eccentricities, which were used to generated artificial RV data. The red curve is the Beta distribution using the MAP solution of a and b. The **black curve** is the **real underlying Beta distribution Beta(0.867,3.03)**. Our derived a and b agree with the real values very well.

3. *Bayesian Model Comparison*

For Nested models: Savage-Dickey ratio

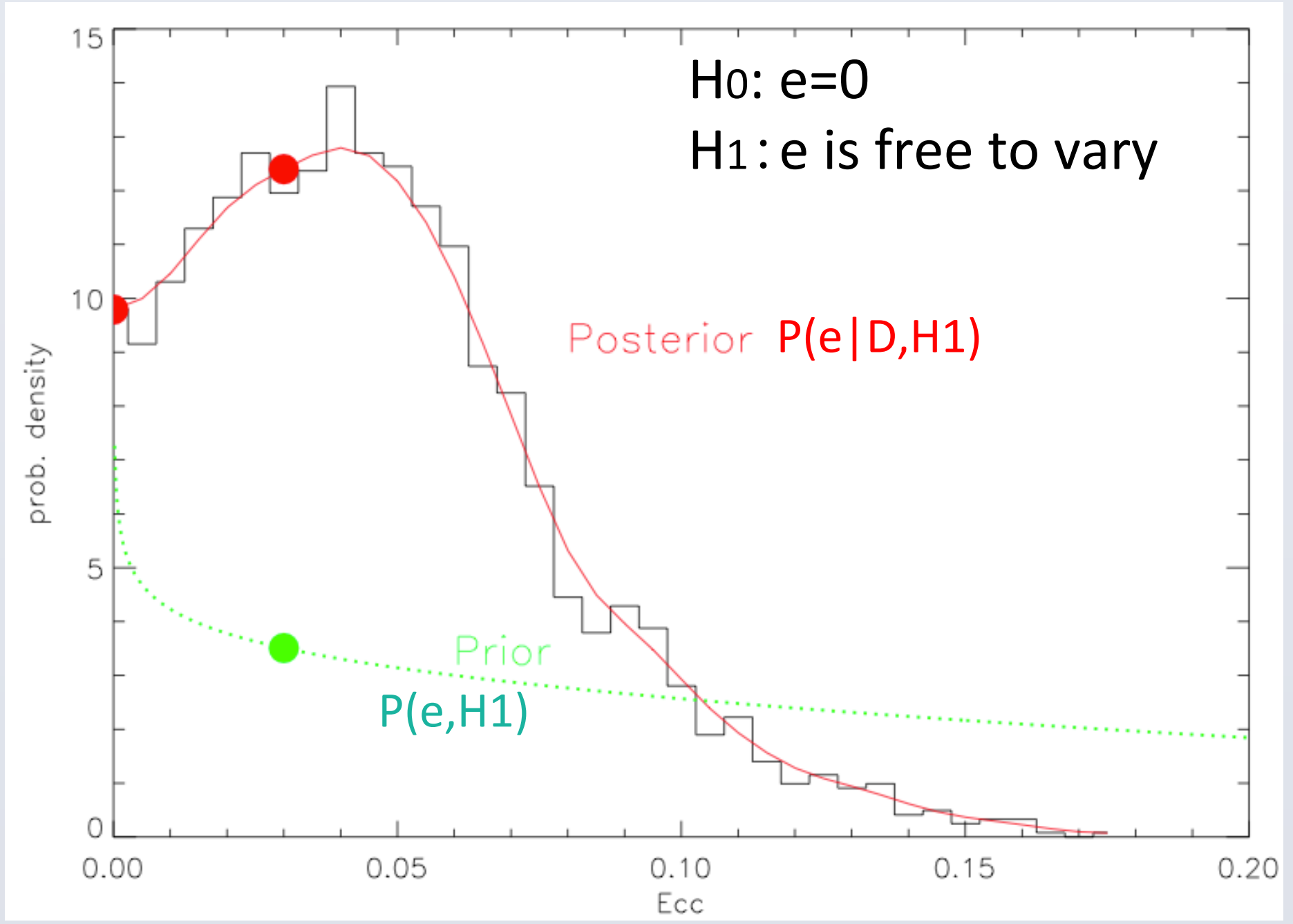
H0: Null hypothesis (parameter is fixed to a pre-specified value

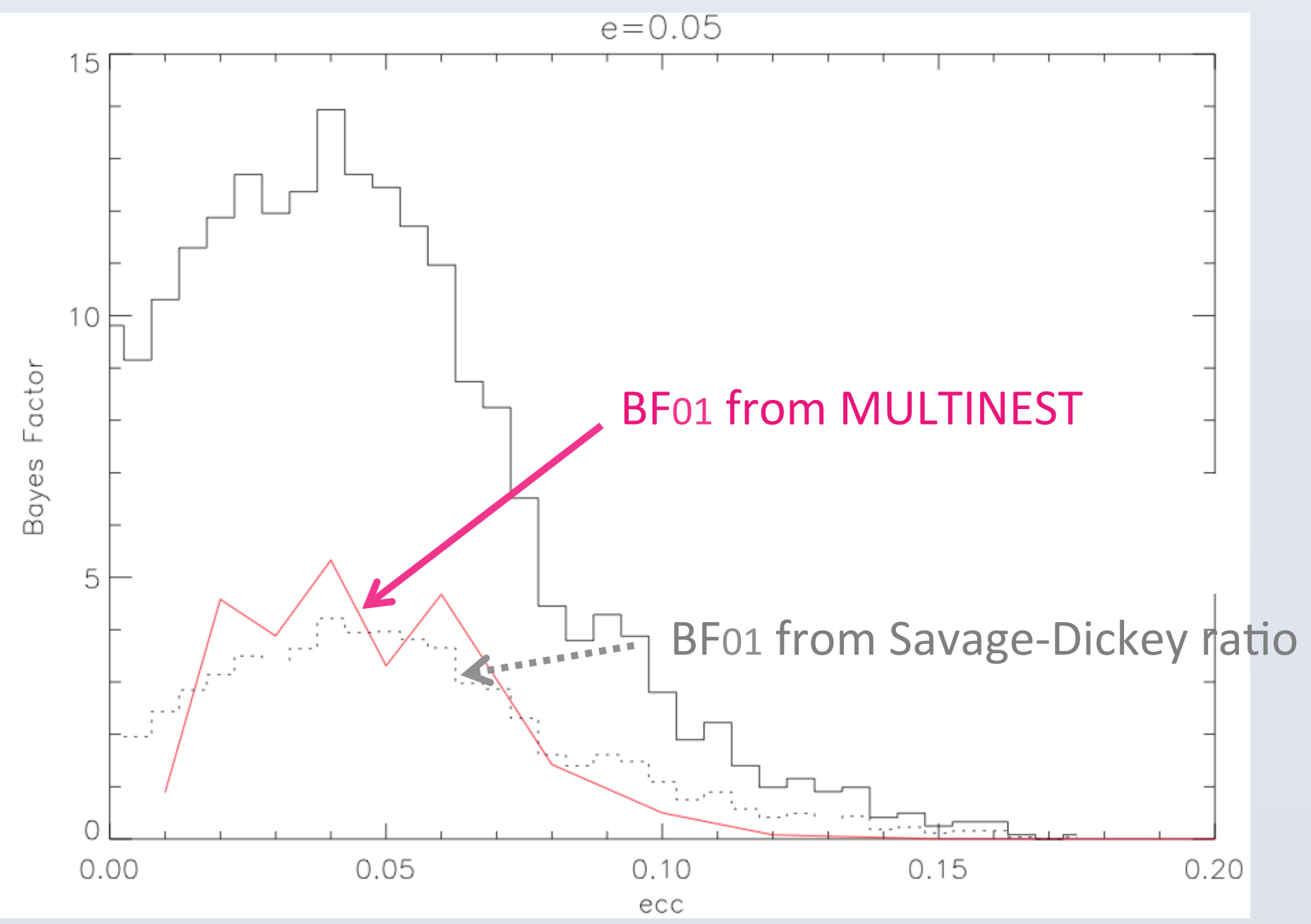
H1: Alternative hypothesis (parameter is free to vary)

posterior

$$BF_{01} = \frac{p(D \mid \mathcal{H}_0)}{p(D \mid \mathcal{H}_1)} = \frac{p(\phi = \phi_0 \mid D, \mathcal{H}_1)}{p(\phi = \phi_0 \mid \mathcal{H}_1)},$$

prior





The negative log-likelihood of a sample of 0 from Poisson(phi) is phi

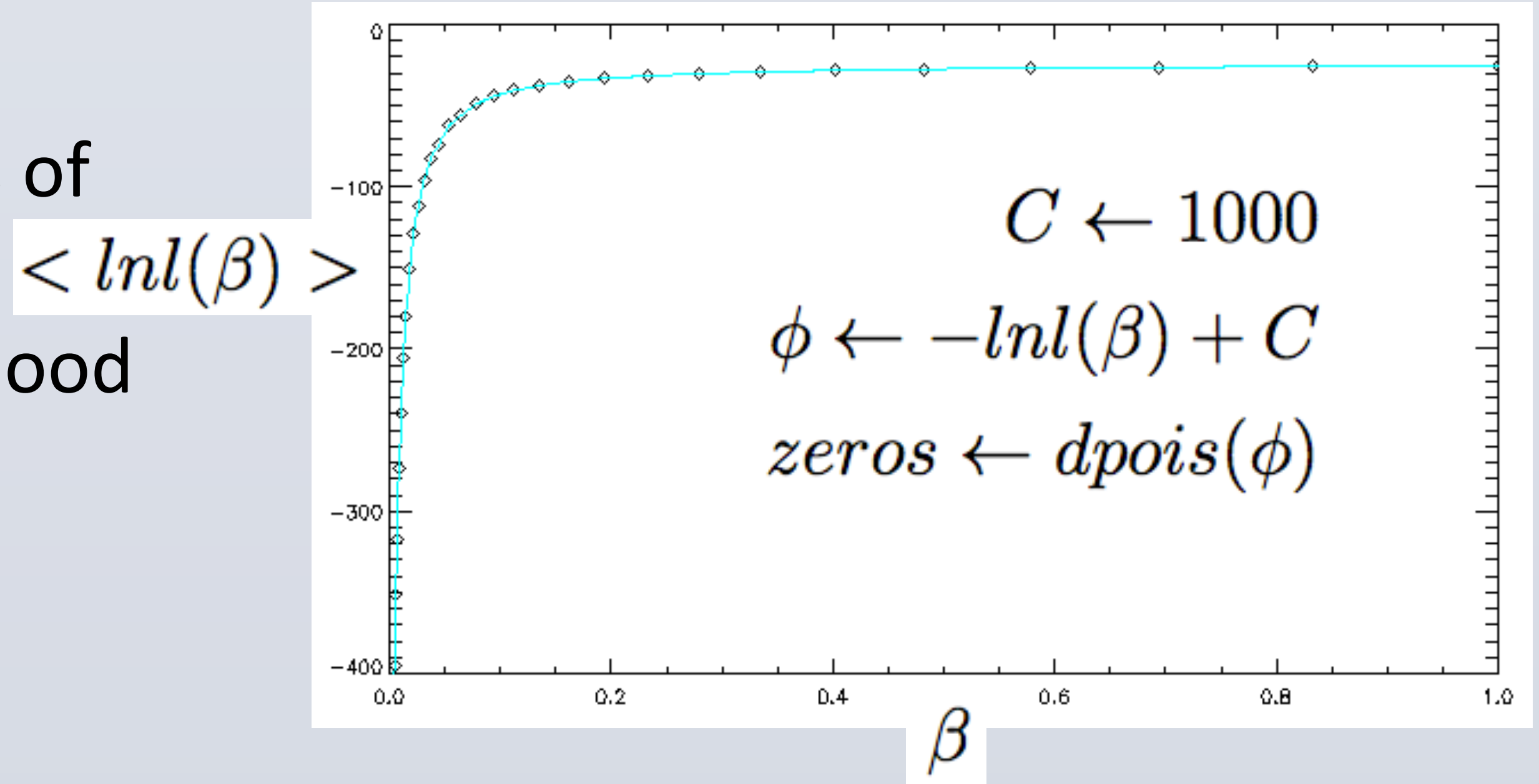
More general: Use tempered posteriors

1 Sample the tempered posteriors for a series of temperatures β

2 Evaluate the average value of original likelihood with the above samples for each β

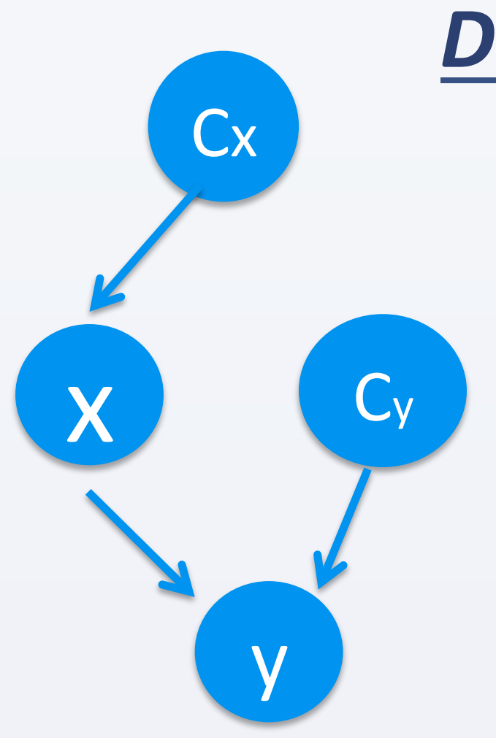
3 Evaluate integral $\ln Z(\beta=1) = \int_0^1 d\beta \langle \ln l \rangle_\beta$

Implemented in JAGS: “zeros trick”

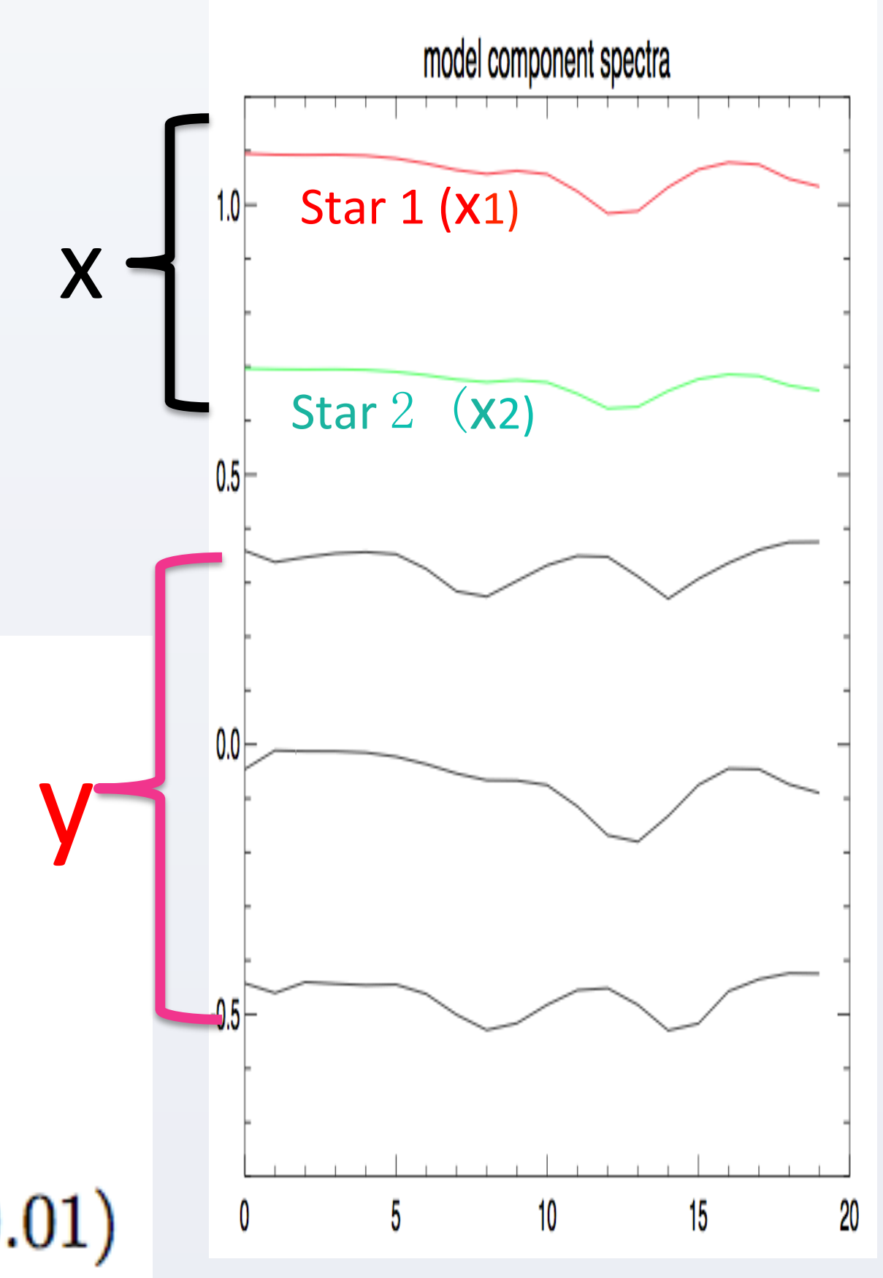


$$C \leftarrow 1000$$
$$\phi \leftarrow -lnl(\beta) + C$$
$$zeros \leftarrow dpois(\phi)$$

4. *Separate the Composite Spectra of Double Line Binaries*



$$y \sim N(y|Ax, C_y)$$
$$x \sim N(x|\mu, C_x)$$
$$C_y = I_{\sigma_y^2}$$
$$C_x = I_{\sigma_x^2}$$
$$\sigma_x \sim Gamma(0.01, 0.01)$$
$$\sigma_y \sim N(\sigma_y|\mu_{\sigma_y}, 0.01)$$



The spectral lines of binaries are blue/red shifted due to Doppler motion of stars.
We want to reconstruct the individual spectrum for each star (**x**=[x1,x2]) from a series of observed double-line spectra (y). This is a linear inverse problem: y=A**x**
(Solve for x, given A and y, A is the matrix of RVs.)

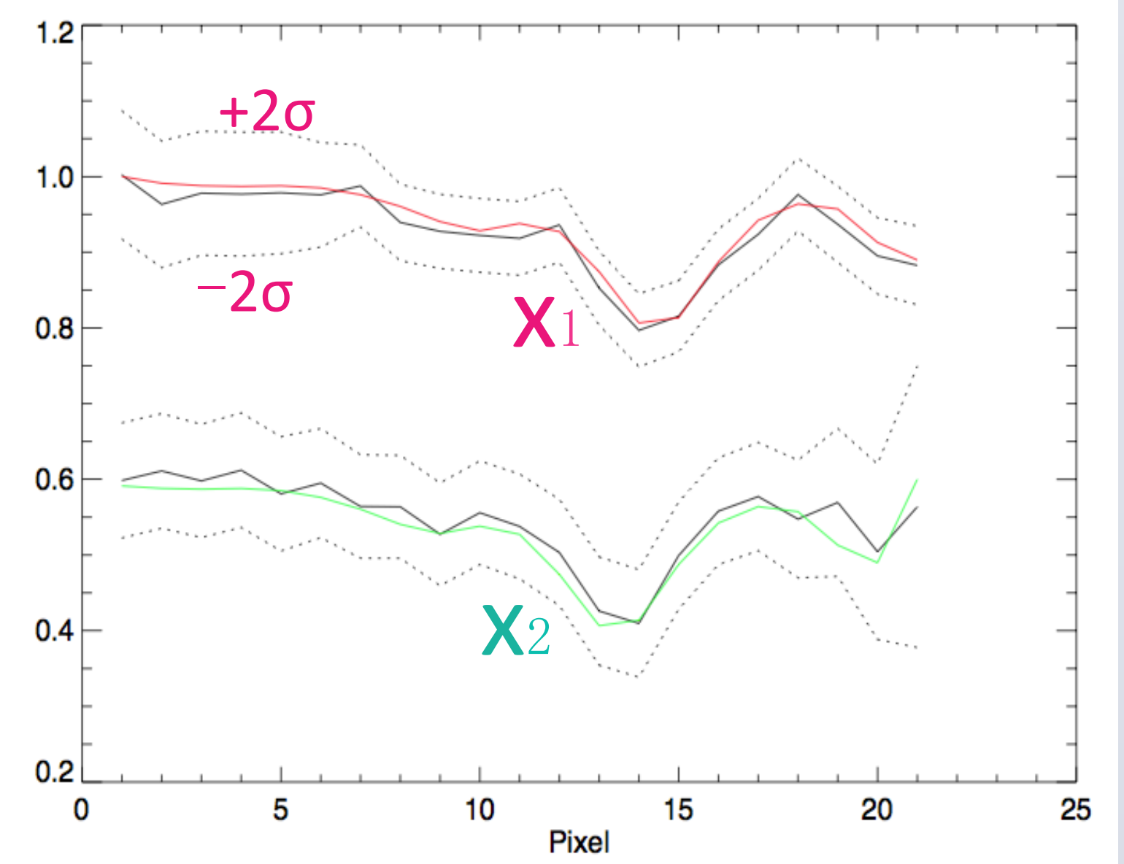
To take into account the pixel correlations, more complex covariance matrix can be used, e.g.

Tikhonov Regularization in Bayesian Paradigm

$$C_x = \sigma_x^2(LL^T)^{-1} \; C_y = \sigma_y^2(LL^T)^{-1}$$

$$\mathbf{L_D} = \begin{bmatrix} -2 & 1 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & & 1 & -2 \end{bmatrix} \quad \mathbf{L_E} = \begin{bmatrix} \delta_0 & 0 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & & 0 & 2\delta_n \end{bmatrix}$$

Zero boundary Free boundary



Reconstructed spectra: (posterior of x)

While this problem can be easily solved by Bayesian Linear Regression, Cx (the covariance matrix of x) is not known and is an regularization parameter. We set hyper-prior on the elements of Cx, thus naturally select the appropriate regularization.