Realistic global simulations of stellar interiors

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JOREK seminar

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Georgia State University
Physics and Astronomy

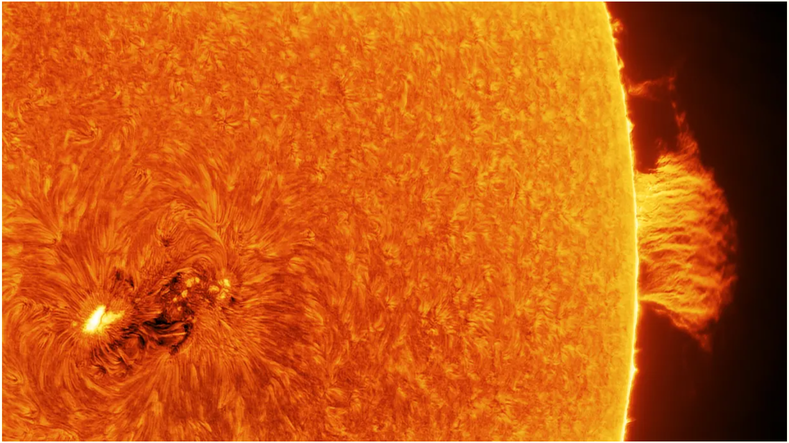
Outline

1. Motivation for studying stars.
2. Stars: what do they look like, and how do we know?
3. Physical models for stars.
4. Numerical set-up & challenges.
5. Transport processes in stars.
6. Big ideas for future work with simulations.

Motivation for studying stars: large range of stellar colors, sizes, and luminosities

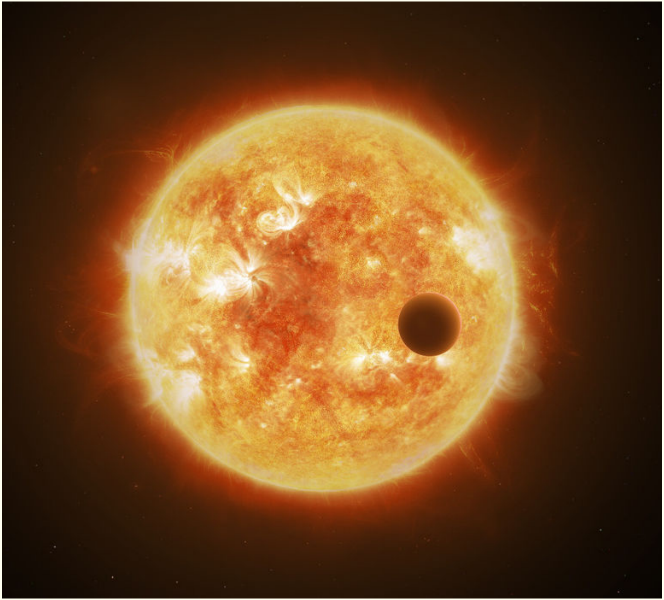
from the Guardian's Astronomy Photographer of the Year 2018 shortlist
Image Credit: Jez Hughes/National Maritime Museum

Practical considerations: space weather and security



from the Guardian's Astronomy Photographer of the Year 2018 shortlist
Image Credit: Łukasz Sujka/National Maritime Museum

Exploration: habitable planets in other solar systems

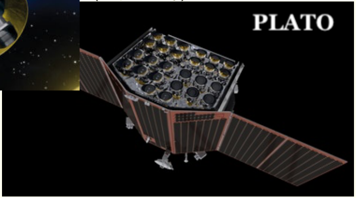


Artwork: the standard method of searching for exoplanets (extra-solar planets).
Image Credit: ESA/ATG medialab.

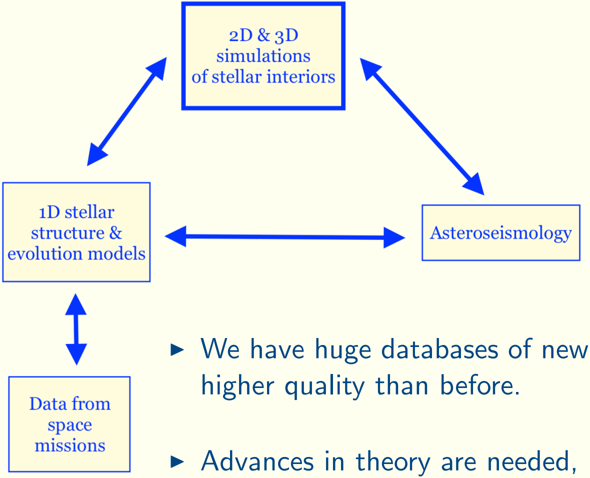
Astronomers are measuring what is out there

- NASA's Transiting Exoplanet Survey Satellite (TESS) launched recently (summer 2018), European Space Agency's GAIA mission for making a three-dimensional map of the Milky Way launched in 2013 and still in operation, and PLAnetary Transits and Oscillations of stars (PLATO) mission, that will be launched in 2026.
- Relationships between the mass of a star, its surface temperature, chemical composition, and luminosity, as well as waves on the surface.

Image Credits: ESA, MIT, and ESA (respectively).



Interpreting the data, and understanding what we cannot directly measure



- We have huge databases of new data, that is higher quality than before.
- Advances in theory are needed, particularly to make sense of these measurements on the surface of stars.

Solar structure

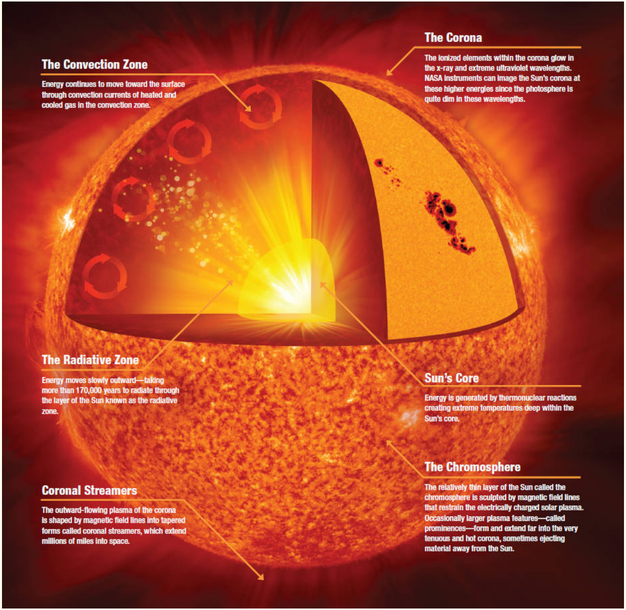
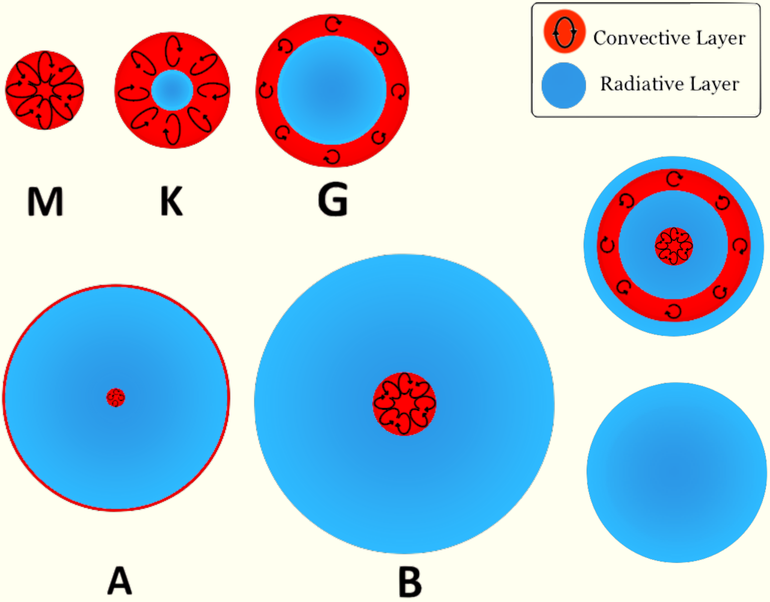


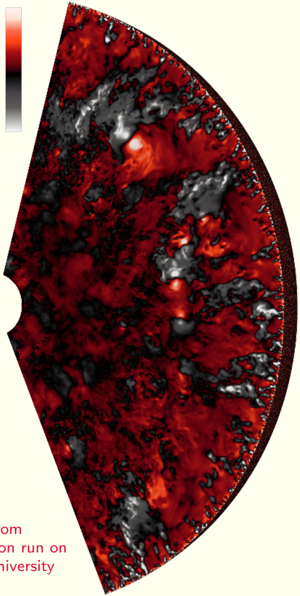
Image Credit: NASA/Jenny Mottar.

Stellar structure: various and changable



Realistic stellar simulations with MUSIC

- Fully compressible hydrodynamic convection with the MUlti-dimensional Stellar Implicit Code (MUSIC).
- Structure, equation of state, and opacities produced using MESA (Modules for Experiments in Stellar Astrophysics) stellar structure and evolution code.
- Goal: to provide a detailed look at the hydrodynamics of the stellar interior.



radial velocity from MUSIC simulation run on Blue Waters (University of Illinois at Urbana-Champaign and the NCSA)

Modeling convection

1. Boussinesq
   - neglects differences in density except in the buoyancy force and equation of state.
2. Anelastic
   - assumes flows remain subsonic.
   - assumes that fluctuations are small relative to a mean state, e.g. $\delta\rho/\rho_0 \ll 1$. In practice, the mean state is updated at intervals.
3. Compressible
   - Density spans a wide range: $10^{-9} \lesssim \rho \lesssim 10^2$ g/cm^3.
   - Stellar Mach numbers span a wide range: $10^{-8} \lesssim \mathrm{M_s} \lesssim 1.0$.
   - In practice we can simultaneously resolve a flow with a range of Mach numbers from 10^{-6} to 0.3.

Fully compressible convection

density
$$\frac{\partial}{\partial t}\rho = -\nabla \cdot (\rho \boldsymbol{u})$$

momentum
$$\frac{\partial}{\partial t}\rho \boldsymbol{u} = -\nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) - \overset{\text{gas pressure}}{\nabla p} + \overset{\text{gravity}}{\rho \boldsymbol{g}}$$

internal energy
$$\frac{\partial}{\partial t}\rho e = -\nabla \cdot (\rho e \boldsymbol{u}) + p\nabla \cdot \boldsymbol{u} + \nabla \cdot (\overset{\text{thermal conductivity}}{\chi} \nabla \underset{\text{temperature}}{T}).$$

- These are hydrodynamic equations. Add-ons for other physics.
- Equation of state used for stellar evolution (partial ionization, electron degeneracy, realistic chemical mix for the sun: H, He, C, N, O, Ne, Mg, Si).
- Thermal conductivity χ is calculated from a realistic opacity for a star.

What about magnetic fields?

- Only 5-10% of stars have visible evidence of magnetic activity (such as star spots).
- Because stars have very long lifetimes, it is necessary to model their structural changes in 1D.

Artist's rendering of possible large star-spots on Betelgeuse, a star in the constellation Orion. Image credit: MPIA Graphics Dept.

- We don't have a good 1D model for how magnetic fields affect density and temperature stratification in stars.
- MHD simulations of stars have this fundamental inconsistency or inaccuracy in them.
- We still *do* MHD modeling of stars (we couple an induction equation) using structures produced without magnetic fields, but MHD simulation is more theoretical than practical at this point.

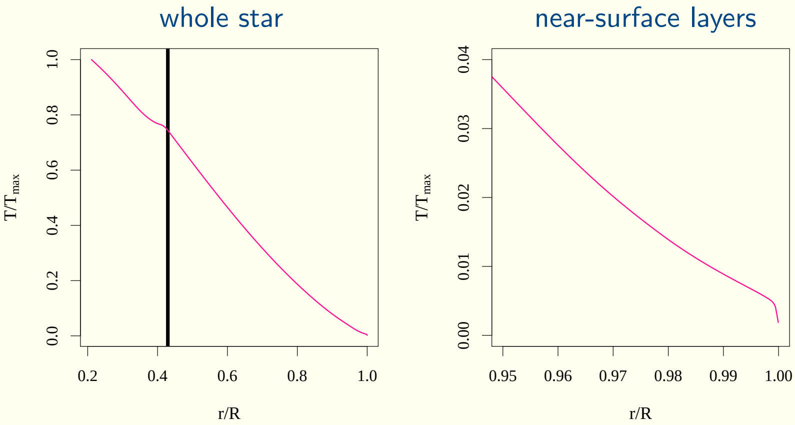
Thermal conductivity and opacity

- The thermal conductivity is related to the temperature and density: $\chi = 16\sigma T^3/3\kappa\rho$.
- σ is the Stefan-Boltzmann constant, from the Stefan-Boltzmann law for black body radiation $\ell = \sigma T^4$ (ℓ is energy radiated per surface area).
- Opacity: absorption and scattering of radiation in a medium.
- κ is the Rossland opacity: average opacity (absorption and scattering of radiation in a medium) weighted with a temperature derivative of the Planck distribution.
- Rossland opacity is valid when the plasma is locally in thermal equilibrium – a good approximation for an optically thick medium like a stellar plasma.
- For the range of temperatures in our young sun, we use tables produced by the Opacity Project at Livermore (OPAL).

Equation of State

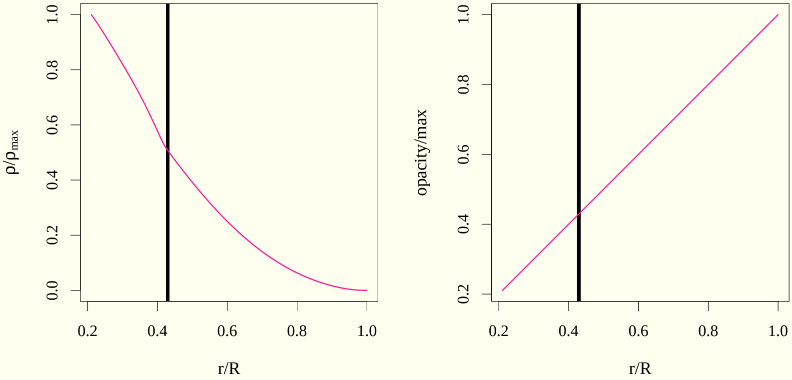
- A typical equation of state for hydrodynamic simulation is: $\boldsymbol{P} = (\gamma - 1)\rho e$, where $\gamma = c_p/c_V = 5/3$ and e is the specific internal energy.
- In a star γ varies due to partial ionization and radiation pressure.
- The degree of ionization is high in the core, but decreases as the surface of the star is approached.
- 1D stellar evolution calculations typically take this into account. Our goal is to do this similarly accurately.
- Pressure $p(\rho, e)$ and temperature $T(\rho, e)$ are interpolated from a tabulated equation of state.
- The tables we use for the equation of state assume: inter-particle forces can be neglected, partial ionization, electron degeneracy, realistic chemical mix for the sun: H, He, C, N, O, Ne, Mg, Si.

Nonlinear temperature gradients from a stellar evolution calculation



Different from the Rayleigh-Bénard convection situation, and different from the box-in-a-star ideal model: no solid boundaries, large height of simulation, non-linear temperature gradient.

Changing opacity and nonlinear stratification in density from a stellar evolution calculation



Density stratification, and opacity/thermal conductivity/thermal diffusivity also have nonlinear stratification.

Numerical models for our stellar simulations

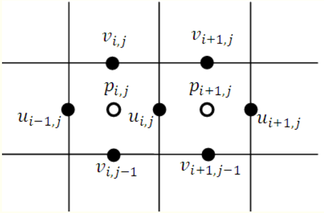
- As you might imagine, there's a lot of overlap between JOREK numerics and MUSIC numerics.
- Our equations are discretized with the Crank-Nicholson scheme (second order).
- Particularly at low Mach numbers, the compressible hydrodynamic equations become stiff.
- "Physics-based preconditioning": the matrix is a solution to our physical system, calculated using an efficient but comparatively inaccurate semi-implicit scheme (acoustic waves and thermal diffusion are treated implicitly, and advection is treated explicitly).
- Low-storage Jacobian-free Newton Krylov (JFNK) solver, fully implicit time integration. (*Knoll & Keyes JCP 2004.*)
- Krylov methods require only a matrix vector product. JFNK methods calculate this using a finite difference approximation without ever explicitly forming the matrix. (Trilinos Library)

Details of the Jacobian Free Newton Krylov solver

- JFNK methods use nested iteration at each time step:
  1. loop for Newton-Raphson iterations
  2. loop over Krylov subspace (GMRES)
  3. loop to determine preconditioner (GMRES)
- Linear iterative method on the inside (loop 3). Picard linearization is used. Linearization errors in calculating the preconditioner should not affect the ultimate solution.
- Nonlinear iterative method on the outside (loop 1), low error compared to semi-implicit methods.
- The details of this are taken care of by a series of packages in the Trilinos library (Sandia National Labs, US).

A Standard Finite Volume Method

velocity $= \langle u, v \rangle$



- staggered grid: scalars are located at cell centers, but velocity components are located at cell boundaries
- simplifies the calculation of hydrostatic equilibrium: $\nabla p = -\rho \boldsymbol{g}$
- Physical quantities are interpolated to grid interfaces using a *monotone upwind scheme for conservation laws* (MUSCL) method.
- Linear reconstruction of variables is limited with a van Leer limiter (1974).
- The resulting finite volume method is second-order accurate in space, total variation diminishing (TVD).

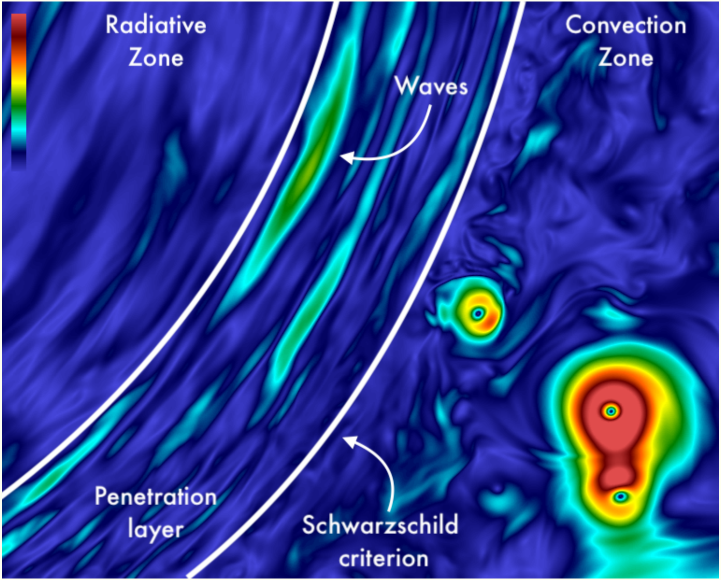
Transport Processes in Stars

- Astronomers can observe directly several quantities: luminosity, and abundances of chemical species on the surface of stars.
- In many cases they can also calculate mass, radius, rotational velocity, and even the internal position of convection zones (based on asteroseismology).
- The *big question* then is age! Astronomers would like to know whether two stars are the same age, and what phase of evolution the star that they just observed is in.
- This is where theory is needed, to produce and improve calculations of the evolution of a stellar structure (density vs radius, temperature vs radius, etc).

Parameterizations and Superparameterizations

- Stellar evolution calculations are done in 1D, necessitating parameterizations of fluid effects into formulas for transport in the radial direction.
- At this point in history, there are hundreds of different competing 1D parameterizations (for waves, convection, rotation, circulations, chemical effects, etc). Each one has free parameters (usually several). The combination of such parameterizations is called a superparameterization.
- Right now our superparameterizations are based on studies of transport processes in isolation.
- We need to study transport processes *in combination* in realistic environments, and then modeling them in 1D parameterizations. This is the largest need in this field: the 321D Link.

Transport across the convective boundary

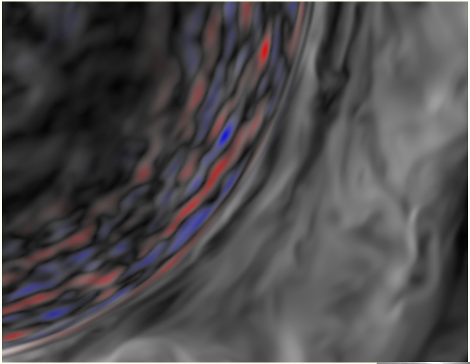


Visualization of velocity magnitude produced with MUSIC.

Convective Boundary Mixing

- The convective boundary is perhaps the most complex region of a star.
- Plumes from the convection zone overshoot into this region.
- Waves produce large shear flows. (Internal gravity waves)
- Differential rotation can cause shear flows, and the rotational profile can change plume dynamics.
- Large-scale circulations can be present.
- Layers of the star have different chemical abundances.
- The relative impact of these different effects changes in different stars. My current work addresses two settings: red giants (the first dredge-up) and binary stars.

A few citations:
Pratt, J., et al. Comparison of 2D and 3D compressible convection in a pre-main sequence star. Astronomy & Astrophysics 638 (2020): A15.
Pratt, J., et al. Extreme value statistics for two-dimensional convective penetration in a pre-main sequence star. Astronomy & Astrophysics 604 (2017): A125.
Pratt, J., et al. Spherical-shell boundaries for two-dimensional compressible convection in a star. Astronomy & Astrophysics 593 (2016): A121.

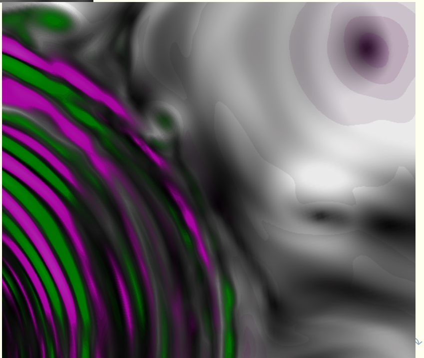


Heat fluctuations in the penetration layer

←

Density fluctuations in the penetration layer

→



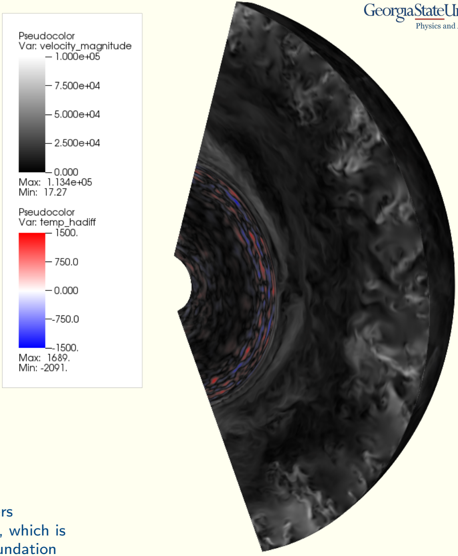
Some final thoughts about stellar physics...

What is important for progress to be made?

1. global simulations, coupling of different layers, including the near-surface layers
2. boundary conditions on the convection zone, including physics of the penetration layer, surface
3. compressible treatment: large density fluctuations + thermal fluctuations in the penetration layer
4. simulations that last long enough so that we can see excitation of internal gravity waves**
5. good radial resolution of the bottom of the convection zone and penetration layer**

** These are sensible reasons for using 2D simulations alongside 3D simulations! The 321D Link needs studies in 2D and 3D.

Thanks!



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