The contained mass is: \( m(a) = 4\pi \int_0^a \rho(a') a'^2 \, da' \).

So the mass element between \( a \) and \( a + \Delta a \) is:
\[
dm(a) = \rho(a) \Delta V_a = 4\pi \rho(a) a^2 \, da
\]

Use (56) & (57) in (58) to find:
\[
dm(a) = \rho \oint_S dm_S = \rho \oint_S g \, dS = \rho \oint_S g \langle g \rangle \frac{4}{3} \pi a^3 \, da
\]

We can rewrite the cons. of mass as:
\[
\frac{da}{dm(a)} = \frac{1}{4\pi a^2 \rho}
\]

Eq. (9) means we can write
\[
\frac{dP}{dm(a)} = -\rho \frac{dP}{dm(a)}
\]

Using (61) we are left with:
\[
\frac{dP}{dm(a)} = \frac{G m(a)}{4\pi a^4} \frac{f_p}{f_p}
\]

with
\[
f_p(a) = \frac{4\pi a^4}{G m(a) a^2} \langle g \rangle - 1
\]

\[N.B. \text{ for a spherical star, } a = r \text{ and } f_p(a) = 1.\]

Further assume our rotating star is in thermal equilibrium. Let \( \delta (a) \) denote the energy passing /s through a level surface in the outward direction, then the increment the luminosity gains (\( \delta (a) \)) bet. \( T \) and \( T + \Delta T \) is:
\[
\delta (a) = \varepsilon \, dm(a) \quad [\text{Ignore } E_x \text{ & } E_g \text{ terms}] \quad (65)
\]
For a chemically homogeneous star, \( \varepsilon = \varepsilon (\rho, T) \) is constant \( \varepsilon \Rightarrow \)
\[
\frac{d\varepsilon}{dm} = \varepsilon
\]

where we suppress a as the radial rotation variable.

Now consider our star in radiative equilibrium \( \Rightarrow (60) \Rightarrow \)
\[
F_a = -\frac{4ac}{3kappa} T^3 \frac{dT}{dn} = -\frac{4ac}{3kappa} T^3 \frac{dT}{dn} \quad (67)
\]

\[N.B. \text{ radiative flux varies } \propto g \text{ on a level surface.}\]
Because, as we've shown in (26), meridional motions carry no net energy, we can write:

\[ \frac{\partial}{\partial t} \left( \frac{3}{5} F \right) \cdot dS = -\frac{24\pi c T^3}{3K} \int_{-\infty}^{\infty} \left( g \times g^{-1} \right) \frac{dt}{dm}, \quad (68) \]

as \( K \) is also const. over surfaces with \( \psi \) const. Use (58) and (68) to find:

\[ \frac{d\psi}{dm(a)} = \frac{-3K}{4\pi c T^3} \left( \frac{d\psi}{dm(a)} \right)^2 f_T \]

where \( f_T(a) = \left( \frac{4\pi a^2}{5a} \right)^2 \left( \frac{\langle g \rangle^{-1}}{\langle g^{-1} \rangle} \right)^{-1} \)

and \( f_T = 1 \) if \( \# \) rotation.

Finally, in regions where convective transport dominates, one usually assumes \( \Gamma_2 \) is constant on level surfaces, so:

\[ \frac{d\psi}{dm(a)} = \left( 1 - \frac{1}{\Gamma_2} \right) \frac{T}{P} \frac{dT}{dm(a)} \quad (71) \]

**Note:** (61), (62), (65) & (69) or (71) REPLACE USUAL EQUATIONS OF STELLAR STRUCTURE. Still need constitutive relations.

All we need are the purely geometrical \( f_p(a) \) & \( f_T(a) \) based on shape of level surfaces.

A full solution would solve:

\[ \nabla^2 \psi = 4\pi G \rho - 2\Omega^2 \]

w/ approx. BC's slightly modified. Usual approach is to approx. total pot, \( \psi(a) \) as soln. for Roche model (3) so that:

\[ \psi(a, \mu) = a \left[ 1 - \frac{\alpha^2}{36\mu} \right] P_2(\mu) \]

or \( \psi(a, \mu) = \frac{G M}{a} - \frac{1}{2} \Omega^2 a^2 (1 - \mu^2) \)

This method is very good \& uniformly rotating centrally condensed stars.

**N.B.** \( P_0(\mu) = 1 \), \( P_1(\mu) = \mu \cos \theta \), \( P_2(\mu) = \frac{1}{2} (3 \cos^2 \theta - 1) \)

In general, \( P_n(\mu) = \frac{1}{2^n n!} \frac{d^n (\mu^2 - 1)^n}{d\mu^n} \).

\[ e.g. P_4(\mu) = \frac{1}{4} \left[ 5 \mu^4 + 10 \mu^3 - 6 \mu^2 - 6 \mu + 1 \right] \]
\[ y = \left( \frac{\Omega^2}{GM} \right)^{1/3} a \]