The point value for each question is given in brackets.

1. Aberration refers to the fact that the direction of travel of a light ray depends on the motion of the observer. Hence, if a telescope observes a star at an inclination $\varphi'$ to the horizontal, then show that classically the 'true' inclination $\varphi$ is given via:

$$\tan \varphi' = \frac{\sin \varphi}{\cos \varphi + \nu/c},$$

where $\nu$ is the velocity of the telescope relative to the star. Show that the corresponding relativistic formula is

$$\tan \varphi' = \frac{\sin \varphi}{\gamma(\cos \varphi + \nu/c)}.$$ 

What is a reasonable range of values for $\nu$ here on earth? Why? [20]

2. Two $\gamma$-rays of different energies collide in a vacuum, yielding an $e^+e^-$ pair. For what ranges of energies of the two $\gamma$-rays and for what range of angles between their initial directions can this pair-creation occur? [25]

3. A space traveler $A'$ travels through space with uniform acceleration $g$ (for maximum comfort). Find the distance covered in 24 years of $A'$s time. [Hint: first express $g$ in units where time is measured in years and distances in light years.] On the other hand, $A''$ describes a straight double path, from location X to Z and back to X, via location Y (i.e., XYZYX) with acceleration $g$ on the XY and ZY legs, and decelerations of equal magnitude on the YZ and YX legs for six years each. Draw a space-time diagram as seen from the Earth, and find by how much the Earth would have aged in 24 years of $A''$s time. [25]

4. How fast would you have to drive towards a red traffic light for the light to appear green so that you could legitimately tell the officer who stopped you (probably with a traction beam) that it was OK to go through? With a speed limit of 50 km/hr and a fine of $10/(km/hr)$ over the limit, how much would you owe the traffic court? [10] (Bonus point: what fraction of the U.S. national debt would you be paying off?).

5. Prove that the Kronecker delta, $\delta^i_j$, is a tensor. What type of tensor is it? [20]