Read §§5.5 and 5.6 and Chapters 4, 7 and §§9.1-9.6 in Kulsrud.

Note that the midterm exam will be given on 8 October and will cover material discussed in class through 26 September; those topics are: basic nature of a plasma, single-particle orbits, bulk fluid equations, waves in a plasma and some material on plasma instabilities.

Each problem is worth 25 points.

1. Show explicitly that for a cold, magnetized plasma we have:

\[ \nabla \times \vec{B}_1 = \frac{-i \omega}{c^2} \vec{\epsilon} \cdot \vec{E}_1 \]

where

\[ \vec{\epsilon} = \begin{pmatrix} \epsilon_1 & -i \epsilon_2 & 0 \\ +i \epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \]

\[ \epsilon_1 = 1 + \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{ce}^2}{\omega^2} + \frac{\omega_{ci}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \]

\[ \epsilon_2 = \frac{\omega_{ce}^2}{\omega^2} \frac{\omega_{ci}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \]

\[ \epsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2} \]

2. Problem 5.3 in Kulsrud

3. Consider the propagation of radio waves through the Earth’s ionosphere, taking account only of the electron component. If \( z \) is the vertical coordinate, suppose that the base of the ionosphere is at height \( z = h_0 \), and that the plasma density increases linearly with height above this. Then the plasma frequency \( \omega_p \) is expressible as:

\[ \omega_p^2 = K(z - h_0), \quad \text{for } z > h_0. \]

a. Calculate the height \( H(\omega) \) at which waves of angular frequency \( \omega \), transmitted vertically from a ground-based transmitter, are reflected from the ionosphere.

b. Calculate the round-trip group travel time for a pulse of wave frequency \( \omega \) which is transmitted vertically, reflected, and returns to its source.

c. Express this time as the equivalent height \( h'(\omega) \) at which, in the absence of the plasma, a horizontal plane reflector would have to be placed so as to produce an echo with the same delay time.

d. Now suppose that, instead of increasing linearly with height indefinitely, the electron density passes through a maximum as some height \( h_m(> h_0) \) and then decreases linearly. Indicate qualitatively the form of \( h' \) as a function of \( \omega \) for this case.

4. Problem 9.2 in Kulsrud.