1. Locate a published refereed paper describing a FR II type radio galaxy (other than Cygnus A) and a second paper describing a FR I type radio galaxy. Give a full citation to each paper and produce a one to two page typed summary of each paper. (Don’t just rephrase the abstract!). Be sure to give key information about each source, such as its: projected linear size, flux, power, distance, and estimated magnetic field strength (perhaps in different parts). Include one or two summary sentences explaining why you think the paper you chose is important or not. A web site at which you might want to begin your search is: adsabs.harvard.edu [24]

2. An ultrarelativistic electron of energy \( E = \gamma mc^2 \) emits synchrotron radiation. Show that its energy decreases with time according to:

\[
\gamma = \frac{\gamma_0}{(1 + A\gamma_0 t)}, \quad \text{with} \quad A = \frac{2e^4B_\perp^2}{3m^3c^5},
\]

where \( \gamma_0 \) is the initial value of \( \gamma \) and \( B_\perp = B \sin \alpha \), with \( \alpha \) the angle between the particle’s velocity and \( B \). [6].

b. Show that the time for the electron to lose half its energy is

\[
t_{1/2} = (A\gamma_0)^{-1} = \frac{5.1 \times 10^8}{\gamma_0 B_\perp^2} \text{s}.
\]

Evaluate \( t_{1/2} \) in years for \( \gamma_0 = 10^6 \) and \( B_\perp = 100 \mu \text{G} \), which are reasonable numbers for extragalactic radio lobes [5].

3. Consider a plane electromagnetic wave propagating through a cold plasma, so that at a fixed location, \( E = E_1 \exp(-i\omega t) \), \( B = B_1 \exp(-i\omega t) \), and \( v = v_1 \exp(-i\omega t) \), so that the dispersion relation is simply: \( \omega^2 = \omega_{pe}^2 + k^2 c^2 \).
If we divide the total energy, \( W \) (averaged over a cycle of a propagating wave) into electric, magnetic, and kinetic energy parts, show that:

\[
\bar{E}_1^2 = 4\pi \bar{W}, \quad \bar{B}_1^2 = 4\pi \bar{W} \left(1 - \frac{\omega^2_{pe}}{\omega^2}\right), \quad \text{and} \quad \bar{mv}_1^2 = \bar{w}^2_{pe}/\omega^2. \quad [10]
\]

4. Consider a slab of neutral plasma, with \( y \) vertically upward. The plasma is unbounded in the \( y \) and \( z \) directions, but bounded in the \( x \) direction. There is a gravitational field of strength \( g \) in the \(-y\) direction. A magnetic field of magnitude \( B \) is in the \(+z\) direction, and it is strong enough so that all motion can be analyzed in terms of drift motions. Suppose the plasma has attained a steady-state.

a. How do the electrons and ions drift in the \( x \) direction due to the perpendicular gravitational and magnetic fields? [4]

b. Calculate the rate of accumulation of surface charge on the surfaces of the slab. [4]

c. Find \( E_x(t) \) as it develops from these surface charges. [4]

d. Calculate the time dependent \( \vec{E} \times \vec{B} \) drift motion in the \( y \) direction. [4].

5. Problem 4.9 in Krishan. [8]

6. Consider a particle gyrating in a circular orbit (at non-relativistic speeds) in an effectively uniform magnetic field.

a. Evaluate the volume of space that contains magnetic field energy equal to that of the kinetic energy of the particle. [4]

b. Consider the cylinder that has this volume and also has the same radius as the orbit of the particle. What is the height of this cylinder? Do you recognize that expression? [4]

7. A radio galaxy is observed at a redshift, \( z = 0.570 \), and one of its lobes has an angular diameter of 3.0 arcsec from VLA measurements. You may assume the Hubble constant, \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), and the deceleration parameter, \( q_0 = 0 \), so the simple Hubble law can be used. When its linear polarization is measured, the direction of the \( \mathbf{E} \) vector of that lobe is observed to be: 27.0° at 10.0 GHz, 44.2° at 5.0 GHz, and 94.4° at 2.8 GHz (all angles are measured \( E \) from \( N \)).

a. What is the physical diameter of this radio lobe? [2].

b. What is the rotation measure (RM) for this lobe? [3].

c. What is the orientation of the unrotated \( \mathbf{B} \) field? [2].

d. Assume that any magnetic field between us and the lobe is completely tangled so there is no net component of \( \mathbf{B} \) along the line-of-sight outside the lobe. Further assume a uniform \( \mathbf{B} \) field within the lobe and that the number density of electrons performing the Faraday rotation in the lobe is \( n_e = 7 \times 10^{-6} \text{ cm}^{-3} \). What is the magnitude of \( B_\parallel \) in that lobe? [3].

8. Recall our derivation of the dispersion measure, which led to the time of propagation
of radio waves being given approximately by
\[ t = \frac{d}{c} + \frac{DM}{\nu^2}, \quad \text{with} \quad DM = C \int_0^d n_e ds. \]

a. Derive the coefficient \( C \) in both cgs units and units where \( d \) is in parsecs. (This is just a rehash of the lecture notes, but explicitly show how you plug in the numbers.) [3]

b. The signal from a particular pulsar at 408 MHz arrives 3.876 seconds earlier than the signal at 93 MHz. What is the dispersion measure for that pulsar? [3]

c. If the mean electron density of the ISM in the direction of that pulsar is \( n_e = 0.05 \text{ cm}^{-3} \), what is the distance to the pulsar, in cm and in pc? [2]

9. Now examine a situation similar to that in question (8), but this time include the effects of a magnetic field in the ISM. Consider the special case of electromagnetic waves propagating parallel to the magnetic field.

a. Show that the group velocity, \( v_g \) is:
\[ v_g^{-1} = \frac{1}{c} \left[ 1 + \frac{\omega_p^2 \omega_{ce}/2\omega(\omega \pm \omega_{ce})^2}{1 - (\omega_p^2/\omega(\omega \pm \omega_{ce}))^{1/2}} \right]. \] [8]

b. Use the notation \( \nu_p = \omega_{pe}/2\pi \), etc., and assume that \( \nu_p \ll \nu \) and \( \nu_c \ll \nu \), to get the approximate expression:
\[ v_g^{-1} = \frac{1}{c} \left[ 1 + \frac{\nu_p^2}{2\nu^2} \left( 1 + \frac{3\nu_p^2}{4\nu^2} \pm \frac{2\nu_c}{\nu} \right) \right]. \] [5]

c. Show that the time delay between the reception of signals at frequencies \( \nu_1 \) and \( \nu_2 \) can be expressed as
\[ \Delta t = DM(\nu_1^{-2} - \nu_2^{-2})(1 + T_1 \mp T_2), \]
where
\[ T_1 = \frac{\langle \nu_1^2 \rangle}{\langle n_e \rangle^2} \frac{3DM \nu_1(\nu_1^2 + \nu_2^2)}{2d\nu_1 \nu_2}, \quad \text{and} \]
\[ T_2 = 2\langle \nu_c \rangle \frac{(\nu_2^2 - \nu_1^2)}{\nu_1 \nu_2 (\nu_2^2 - \nu_1^2)}, \]
where the averages are defined by: \( \langle Q \rangle \equiv d^{-1} \int_0^d Q ds \) [6].

d. For \( \nu_1 = 40 \text{ MHz} \) and \( \nu_2 = 430 \text{ MHz} \), observations have shown that the corrections \( T_{1,2} \) are both less than \( 3 \times 10^{-4} \). Use this to obtain upper limits to both DM and B. What are the values of \( \nu_p \) and \( \nu_c \) at those limits? Are they indeed much less than \( \nu_1 \), as assumed in the derivation in (b)? [6].