The resistive MHD Ohm's law

- Thus far we have the resistive MHD Ohm's law

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \tag{9}$$

where resistivity is the mechanism that breaks the frozen-in condition

- The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{10}$$

$\Rightarrow$ resistive diffusion of $\mathbf{B}$

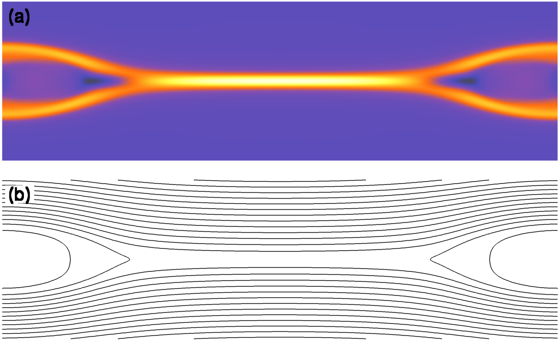
Return of the generalized Ohm's law

- The generalized Ohm's law is given by

$$\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c} = \eta\mathbf{J} + \underbrace{\frac{\mathbf{J}\times\mathbf{B}}{en_e c}}_{\text{Hall}} - \underbrace{\frac{\nabla\cdot\mathbf{P_e}}{n_e ec}}_{\text{elec. pressure}} + \underbrace{\frac{m_e}{n_e e^2}\frac{\mathrm{d}\mathbf{J}}{\mathrm{d}t}}_{\text{elec. inertia}} \qquad (11)$$

- The frozen-in condition can be broken by
  - The resistive term
  - The divergence of the electron pressure tensor term
  - Electron inertia
- The Hall effect doesn't break the frozen-in condition but can restructure the reconnection region
- These additional terms introduce new physics into the system at short length scales
  - Ion inertial length, ion sound gyroradius

Simulation with the Hall term off (resistive MHD)

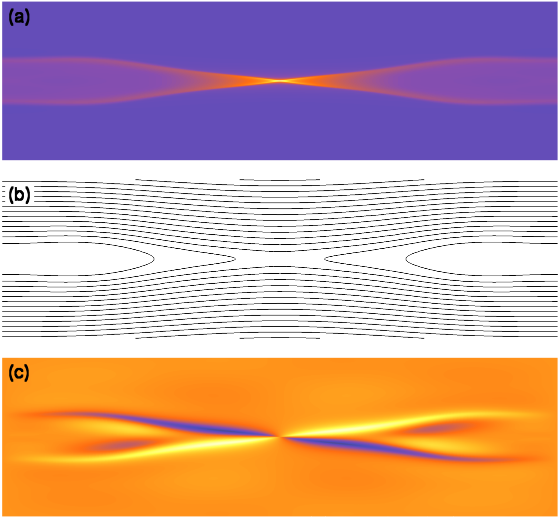


Current density

Magnetic flux

- Elongated current sheet $\Rightarrow$ slow reconnection

Simulation with the Hall term on (Hall MHD)



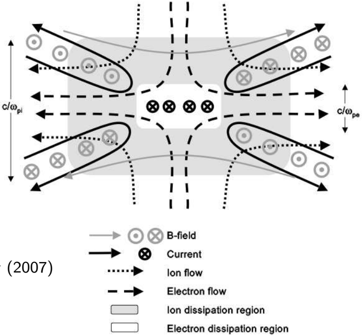
Current density

Magnetic flux

Out-of-plane
magnetic field

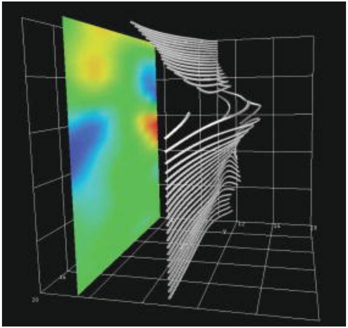
- X-point structure in diffusion region! Fast reconnection! Quadrupole out-of-plane magnetic field!

Fundamentals of collisionless reconnection



Drake & Shay (2007)

Yamada et al. (2006)



- On scales shorter than the ion inertial length, electrons and ions decouple. The magnetic field is carried by the electrons.
- The electrons pull the magnetic field into a much smaller diffusion region
  - $\Rightarrow$ X-point geometry $\Rightarrow$ fast reconnection
- The in-plane magnetic field is pulled by electrons in the out-of-plane direction $\Rightarrow$ quadrupole magnetic field

The Hall effect is not the whole story

- In resistive Hall MHD, elongated current sheets become more like X-points
- The $\frac{\nabla \cdot \mathbf{P_e}}{n_e e c}$ term is best studied using fully kinetic particle-in-cell (PIC) simulations
  - Important area of current research
- PIC simulations of reconnection in a positron-electron plasma still show fast reconnection!
  - Hall term is absent because $m_{e^+} = m_{e^-}$

2D PIC simulations with a large domain show an elongated current sheet with occasional island formation

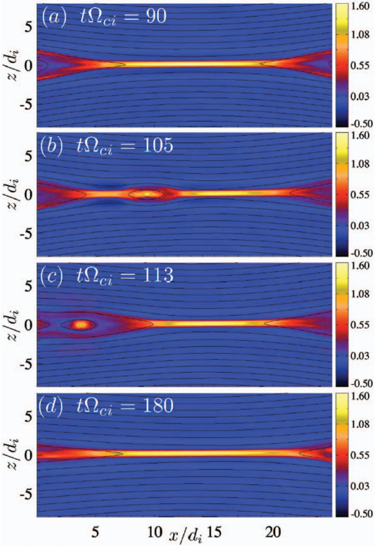
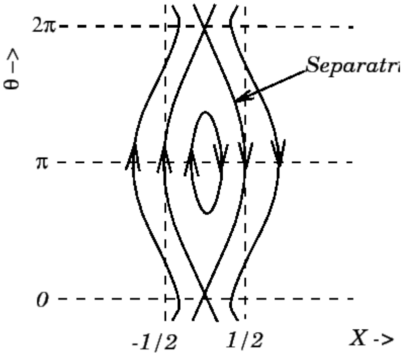


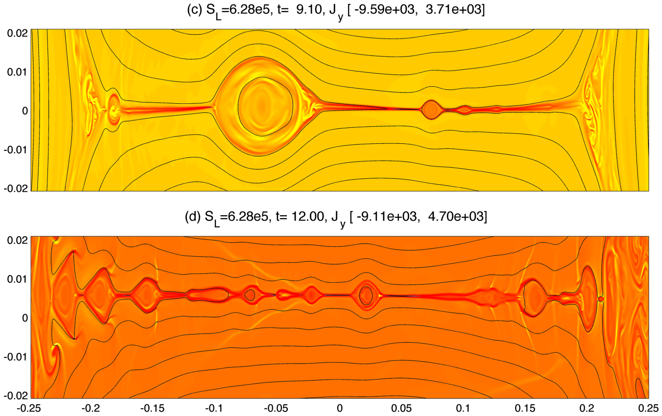
FIG. 9. (Color) Out-of-plane electron velocity U_{ey} at four different simulation times showing the stretching of the electron diffusion region and production of a secondary island. These results are for the $25d_i \times 25d_i$ open boundary case.

The tearing mode is a resistive instability



- The tearing instability breaks up a current sheet into a chain of X-points and magnetic islands
- Use asymptotic matching between inner and outer solutions to calculate exponential growth rate
- Degrades confinement in magnetically confined fusion plasmas

Elongated current sheets are susceptible to the tearing-like plasmoid instability (Loureiro et al. 2007)



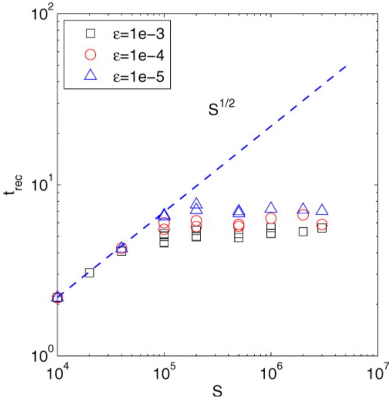
Bhattacharjee et al. (2009)

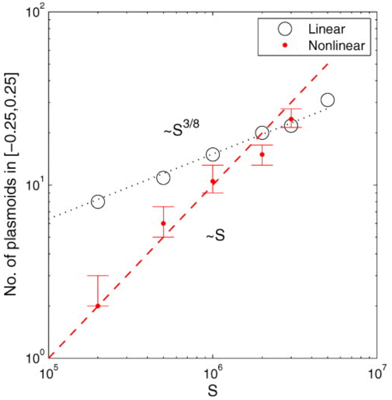
- The reconnection rate levels off at $\frac{V_{in}}{V_A} \sim 0.01$ for $S \gtrsim 10^4$
- The Sweet-Parker model is not applicable to astrophysical reconnection!

Properties of the plasmoid instability

- The linear growth rate scales as $\sim S^{1/4} V_A / L$
  - Instability gets worse with increasing Lundquist number!
  - Number of islands scales as $S^{3/8}$ in linear regime
- The tearing mode scales as $S^{-3/5}$ or $S^{-1/3}$ depending on the regime
  - Growth rate decreases with increasing Lundquist number
- The difference in scaling occurs because the thickness of Sweet-Parker current sheets scales as $\delta \sim S^{-1/2}$

The scaling of the plasmoid instability can be investigated using large-scale 2D resistive MHD simulations





- The reconnection time scale asymptotoes to a roughly constant value above a critical Lundquist number! (left)
  - Fast reconnection occurs even in resistive MHD!

But does the plasmoid instability lead to fast enough reconnection?

- The plasmoid instability predicts $\frac{V_{in}}{V_A} \sim 0.01$
- Reconnection rates of 0.1 are needed to describe flare reconnection
- Shepherd & Cassak (2010) argue that this instability leads to the formation of structure on small enough scales for collisionless reconnection to develop
- The collisionless reconnection then gives the fastest reconnection rates
- What happens in 3D?