Plasma & MHD.   (April 1)   2019

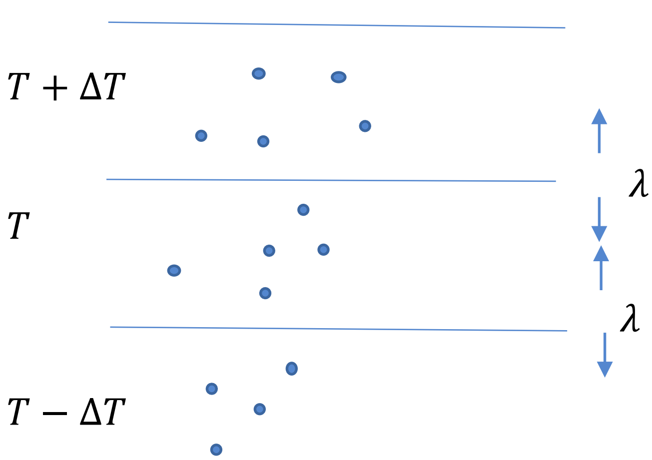
Transport coefficients:
Resistivity, thermal conductivity, viscosity shear and compressive.

Main concepts:

1) $\lambda = mf_p$ = Mean Free Path
2) $V_T(i,e)$ = Thermal velocty $\sqrt{\frac{KT_{e,i}}{m_{e,i}}}$ per dimension.
3) collisoin frequency $\nu_{e,i} = V_T(i,e)/\lambda$
4) mf_p in plasma because of long-range electric field electron interacts with many other charge particles with many other particles. mf_p produced by coulomb logarithm.

5) $\tau = \frac{1}{\nu_{e,i}}$ = collisoion tim. Same for protons and electrons, in P_{0e}collisoionds: λ is not because of protons move much slower.

Now let us derive thermal conductivity from a Back-of-the Envelope approach:



The gradient of $\vec{\nabla} T$ is in the $\hat{z}$ -direction

An electron will travel one mf_p before a collision (on average) with temperature difference $\Delta T = |\vec{\nabla} T|\lambda$. Thus, the electron energy will carry 3/2 $K\Delta T$ in energy.
Thus, the energy flux

$\overrightarrow{F_C} = n_e V_T \Delta\vec{E} = n_e V_T \vec{\nabla} T \lambda$

$$\lambda n A \cong 1$$

A is the cross section for collision.

$$\frac{e^2}{r} = \frac{1}{2} m V_T^2\,, \quad and \quad A = \pi r^2$$

$\Longrightarrow r = \frac{2e^2}{mV_T^2} \quad \Longrightarrow A = \frac{4\pi e^4}{m^2 V_T^4}$

Hence: $\lambda = \frac{m^2 V_T^4}{4\pi e^4 h}$. And,

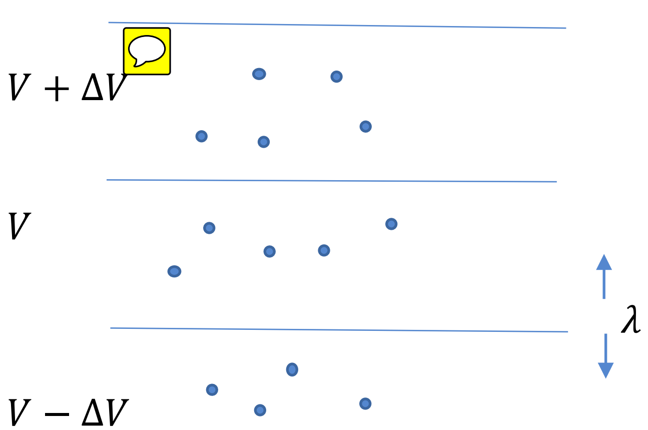
$V_T = \sqrt{KT_e} \quad \Longrightarrow$

$\overrightarrow{F_C} = k_0 T^{5/2} \vec{\nabla} T, \qquad k_0 \approx 10^{-6}$ (canonic)

Heating from conductive heat flux:
No net heating, so the unit flux changes:

$$\mathrm{E} = \nabla . \overrightarrow{F_C}$$

Viscosity, shear:
Same analysis, using mf_p, but now we look at momentum transportation, not energy. Plus, let us first look at shear transport,



$\Delta P_{\perp} = m\left(\Delta \vec{V}_{\parallel}\right)\lambda.$ $\parallel$ is in the $\hat{x}$direction while $\perp$ is in the $\hat{z}$ direction.

The shear momentum flux is then,

$\Delta P_{\perp} = m\left(\Delta \vec{V}_{\parallel}\right)\lambda.$

$\overrightarrow{F_p} = nV_T \Delta P_{\perp} = nmV_T\lambda(\Delta \vec{V}_{\parallel})$

$$\lambda = \frac{m^2 V_T^4}{4\pi e^4 h}$$

Note:
1) As usual n (density) drops out again.
2) Temperature dependence again as $T^{\frac{5}{2}}$.
But how about mass dependence, (protons versus electrons)?
Working out m dependence and assuming $T_e = T_p$

$$\overrightarrow{F_p} = \frac{m^3 V_T^5}{4\pi e^4} \vec{\nabla} V_{\parallel} \& \quad V_T = \left(\frac{KT}{m}\right)^{1/2}$$

Here: $$\overrightarrow{F_p} = \frac{m^{1/2}(KT)^{5/2}}{4\pi e^4} \vec{\nabla} V_{\parallel}$$
$$= \mu_0 T^{5/2} \vec{\nabla} V_{\parallel}$$

Since the momentum flux scales with $m^{1/2}$, this flux is dominated by protons. The viscous force is just as with thermal flux, the divergence of the momentum flux:
$\overrightarrow{F_v} = \vec{\nabla}.(\mu_0 T^{\frac{5}{2}} \vec{\nabla} V),$ $\mu_0 \approx 10^{-17}$(cgs) in canonical plasma.