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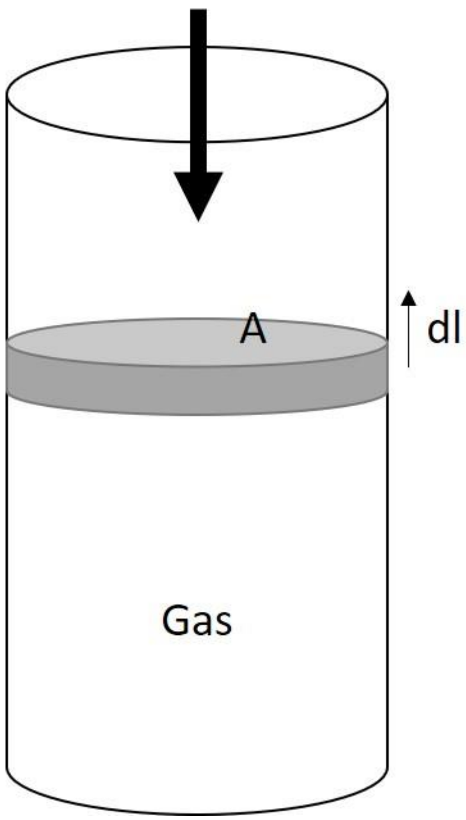
Energy Equation for plasma using the first law of Thermodynamics

1st Law of thermodynamics



$$dQ = dU + dW$$

$$dW = PdV \longrightarrow (1)$$

$$(\because W = \frac{F}{A} A dl = F\, dl)$$

If piston can't move up

$$dW = 0$$

$$dQ = C_V dT$$

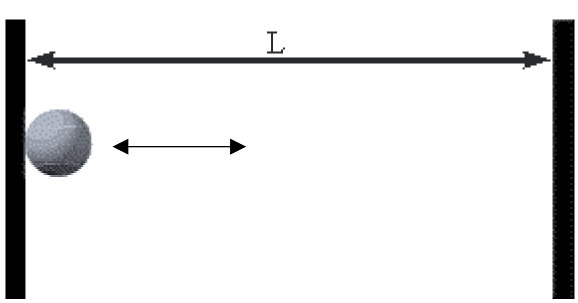
Where C_v is specific heat

For ionizing plasma, the change in kinetic energy of a particle,

$$\triangle E_{K.E.} = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2$$

Isentropic exponent, $\gamma = 1 + \frac{2}{f}$ where f is the degree of freedom

$$E = n\left(\frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2\right) = \frac{3}{2} v_i^2 n$$



$$P = F/A$$

Assume particle has velocity v_x in x the direction, if the mass of the particle is m and the change of the momentum is Δp

$$\Delta p = 2 m v_x$$

Time for successive hits to the same wall is,

$$\Delta t = \frac{2L}{v_x}$$

Then the force exerted on the wall is,

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_x^2}{L}$$

So the pressure from a particle is,

$$P = \frac{F}{A} = \frac{mv_x^2}{AL} = \frac{mv_x^2}{V} \longrightarrow (2)$$

But,

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \Longrightarrow \langle v_x^2 \rangle = (1/3)\, \langle v^2 \rangle \longrightarrow (2)$$

From equation (2) and (2),

$$PV = \frac{1}{3} m \langle v^2 \rangle = \frac{2}{3}\left(\frac{1}{2} m \langle v^2 \rangle\right) = \frac{2}{3} E_{K.E.} \Rightarrow E_{K.E.} = \frac{3}{2} PV$$

Internal energy, $U = E_{K.E.} = (3/2)PV$

$$dU = d\left(\left(\frac{3}{2}\right) PV\right) = \frac{3}{2} VdP + \frac{3}{2} PdV \longrightarrow (4)$$

From equation (1) and (4),

$$dQ = dU + dW$$

$$dQ = \frac{3}{2} VdP + \frac{3}{2} PdV + PdV$$

$$dQ = \frac{3}{2} VdP + \frac{5}{2} PdV$$

$V\rho = N$, # of particles

$\frac{d}{dt}\left(\frac{3P}{2\rho}\right) + \frac{5P}{2\rho} \vec{\nabla}. \vec{v} = 0$ For a monoatomic adiabatic plasma

The rest is continued on the slides…