

Cowling's anti-dynamo theorem

- ▶ In dynamos, too much symmetry is a bad thing
- ▶ Cowling (1934): a steady axisymmetric field cannot be maintained by dynamo action
- ▶ Cowling's theorem applies to exact asymmetry, but in practice successful dynamos cannot possess too much symmetry
- ▶ Suppose we start out with a magnetic field of the form

$$\mathbf{B} = B_\theta(r, z)\hat{\boldsymbol{\theta}} + \mathbf{B}_p(r, z) \quad (3)$$

where B_θ is the toroidal (azimuthal) component and \mathbf{B}_p is the poloidal (radial & axial) component

- ▶ Assume that the magnetic field is time-independent and non-uniform

The poloidal field consists of closed flux surfaces

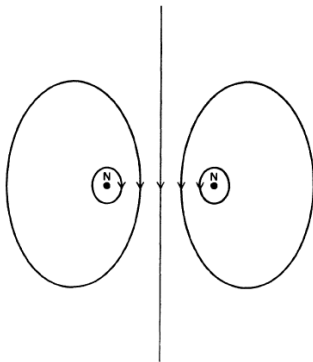


Fig. 9.2. The magnetic field lines in a meridional plane for an axisymmetric field.

- ▶ By symmetry, each meridional half-plane must contain at least one O-type neutral point in the poloidal field (denoted 'N')

Integrate Ohm's law around the closed field line

- ▶ Ohm's law is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \quad (4)$$

- ▶ The path integral along the closed field line (or null line) is

$$\begin{aligned} \oint_C \eta \mathbf{J} \cdot d\mathbf{s} &= \oint_C \mathbf{E} \cdot d\mathbf{s} + \oint_C \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot d\mathbf{s} \\ &= \int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} + \oint_C \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot d\mathbf{s} \end{aligned} \quad (5)$$

where we used Stokes' theorem.

- ▶ The first term on the RHS vanishes because we assume a steady state
- ▶ The second term on the RHS vanishes because \mathbf{B} is parallel to the path integral

Finishing the proof

- ▶ Ohm's law reduces to

$$\oint_C \mathbf{J} \cdot d\mathbf{s} = 0 \quad (6)$$

- ▶ However, the current cannot vanish because closed flux surfaces surround N
- ▶ Therefore, this magnetic field configuration cannot sustain itself by dynamo action!
- ▶ Led to decades of skepticism that dynamo theory could work
- ▶ The difficulty with an axisymmetric field is maintaining the poloidal field that decays due to resistive diffusion along N
- ▶ Progress really began again two decades later with existence theorems for possible dynamo mechanisms (Herzenberg 1958; Backus 1958)