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(1) General Energy equation in Plasma
Use induction + momentum equation

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{V} \times \vec{B} - \eta \vec{J} \right) \quad (1)$$

$$\rho \frac{d\vec{V}}{dt} = \vec{J} \times \vec{B} + \rho \vec{\nabla} \Psi - \vec{\nabla} P + \text{viscous terms} \quad (2)$$

($\vec{J} \times \vec{B}$: Lorentz Force; $\rho \vec{\nabla} \Psi$: gravity; $\vec{\nabla} P$: pressure)

Take the dot product with $\vec{B}$ Form (1)

$$\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = 4\pi \frac{\partial}{\partial t} \left(\frac{B^2}{8\pi} \right) = \vec{B} \cdot \vec{\nabla} \times (\vec{V} \times \vec{B} - \eta \vec{J}) \quad (3)$$

Use, again, the vector relation

$$\vec{\nabla} \cdot (\vec{A}' \times \vec{B}') = \vec{B}' \cdot \vec{\nabla} \times \vec{A}' - \vec{A}' \cdot \vec{\nabla} \times \vec{B}' \quad (4)$$

Hence: $4\pi \frac{\partial}{\partial t} \left(\frac{B^2}{8\pi} \right) = \vec{\nabla} \cdot (\vec{B} \times \vec{E}) + (\vec{\nabla} \times \vec{B}) \cdot \vec{J} \frac{4\pi}{c} - \eta J^2 \frac{4\pi}{c}$ (5)

($\vec{E}$: using Ohm's Law)

Another vector identity: $(\vec{\nabla} \times \vec{B}) \cdot \vec{J} = (\vec{B} \times \vec{B}) \cdot \vec{V}$

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Hence: (work by Lorentz force; Ohmic dissipation; divergence of Poynting flux)

$$\frac{\partial E_m}{\partial t} = -(\vec{J}\times\vec{B})\cdot\vec{v} - \eta J^2 - \vec{\nabla}\cdot\vec{P} \quad (6)$$

where: $\vec{P} = \frac{c}{4\pi}(\vec{E}\times\vec{B})$ is Poynting flux (7)

Now take the dot product with $\vec{v}$ of the momentum equation

$$\frac{\partial E_{kin}}{\partial t} = -\vec{v}\cdot\vec{\nabla}P - \vec{v}\cdot\vec{\nabla}\left(\tfrac{1}{2}\rho v^2\right) - \vec{v}\rho\vec{\nabla}\Psi \quad (8)$$

(pressure; kinetic energy flow; gravity)

$$+\vec{v}\cdot(\vec{J}\times\vec{B}) + \text{Viscous terms}$$

↳ work by Lorentz force

The total energy of the magnetized plasma is the $E_{kin} + E_m$

So, add up (6) and (8) and you find that the Lorentz force term $(\vec{J}\times\vec{B})$ disappears! What is left are Ohmic and viscous dissipation plus gravitational potential and work by pressure plus divergence of $\vec{P}$ and kinetic energy flow

Hence $E_{kin} \xrightarrow{F} E_{mag}$ (mediated by Lorentz force)

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(2) GENERALIZED OHM'S LAW, FOR HW

$$\vec{E} = -\vec{V}\times\vec{B} - \frac{1}{en}(\vec{J}\times\vec{B}) - \frac{\vec{\nabla}P}{en} + \eta\vec{J} + \frac{m_e}{e^2}\frac{\partial}{\partial t}\left(\frac{\vec{J}}{n}\right)$$

- $-\vec{V}\times\vec{B}$: PLASMA MOTION
- $-\frac{1}{en}(\vec{J}\times\vec{B})$: HALL TERM
- $-\frac{\vec{\nabla}P}{en}$: AMBIPOLAR DIFFUSION
- $\eta\vec{J}$: DISSIPATION (COULOMB COLLISIONS)
- $\frac{m_e}{e^2}\frac{\partial}{\partial t}\left(\frac{\vec{J}}{n}\right)$: ELECTRON INERTIA

HW (1): WRITE DOWN THE POYNTING FLUX FOR PLASMAS IN THE IDEAL APPROXIMATION

HW (2): BIERMAN BATTERY. ASSUME A PLASMA DEVOID OF ANY MAGNETIC FIELDS. SHOW THAT IN A FLUID (ρ = CONSTANT), THE PLASMA WILL REMAIN FREE OF FIELDS. THEN SHOW THAT IN A NON-UNIFORM PLASMA-GAS A FIELD CAN BE GENERATED FROM NOTHING!