PHYS 8120 Plasma Physics and Magnetohydrodynamics
Instructor - Prof. Piet Martens
Rezvan Khademhosseini
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Coulomb Logarithm:

We know

$$\tan(\theta/2) = Q = -\frac{e^2}{m_e v_0^2 b}$$

v_0 Is the electron's thermal velocity.

The Spitzer Resistivity in a plasma known from

$$m_e \frac{dv_{e,\|}}{dt} = -eE - m_e \Delta v_{e,\|} v_{i,e}$$

Where $v_{i,e}$ is the electron-proton collision frequency and $v_{e,\|}$

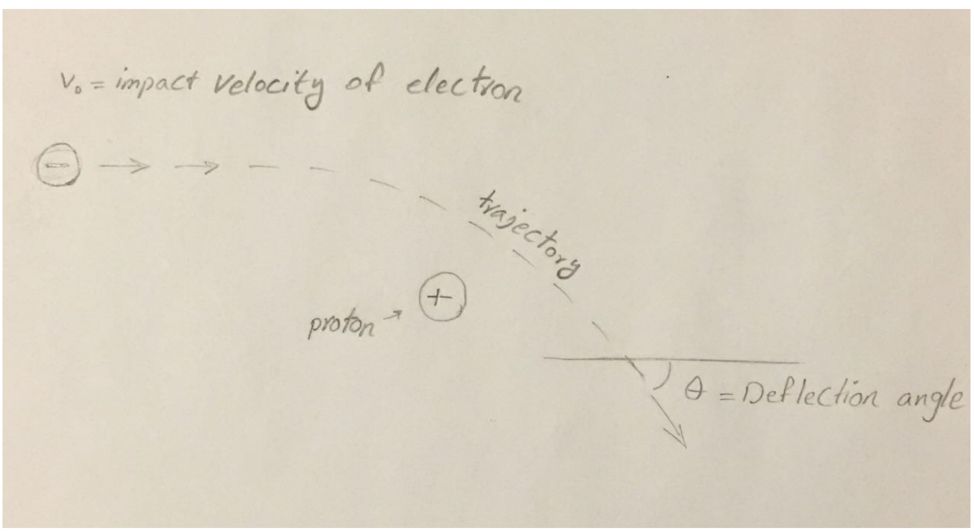
is the velocity of an electron relative to the protons parallel to the electric field, with the current defined as

$$J = -en_e v_{e,\parallel}$$

We find:

$$E = \eta J = -\frac{m_e v_{i,e}}{n_e e^2} \frac{\Delta v_{e,\parallel}}{v_0} J$$

In the zeroth order approach we assume $\Delta v_{e,\parallel} \approx v_0$.



This figure demonstrates:

$$\Delta v_{e,\parallel} = -v_0(1-\cos\theta)$$
$$\Delta v_{e,\parallel} = -v_0 \sin^2(\theta/2)$$

From our orbit calculation, we know electron as a function of b. Assuming an even distribution of incoming electron the probability of an incoming electron having an impact parameter between b and b+db :

$$b + db = 2\pi b db * f$$

$$f = \pi\left(b_{max}^2 - b_{min}^2\right)$$

Which b_{max} and b_{min} determined from physical considerations. We can calculate the average expectation value of $v_{e,\parallel}$:

$$\left\langle \Delta v_{e,\parallel} \right\rangle = -\frac{v_0}{\pi\left(b_{max}^2 - b_{min}^2\right)} \int_{b_{min}}^{b_{max}} \sin^2(\theta(b)/2)2\pi.b.db$$

$$\sin\left(\theta/2\right) = \frac{Q}{\sqrt{1+Q^2}}; \cos\left(\theta/2\right) = \frac{1}{\sqrt{1+Q^2}}$$

$$Q \triangleq -\lambda/b \quad \text{With} \quad \lambda = \frac{e^2}{m_e v_0^2}$$

$$\int_{b_{\min}}^{b_{\max}} \frac{\lambda^2}{b^2} \frac{1}{1+\dfrac{\lambda^2}{b^2}} 2\pi.b.db =$$

$$= \pi\lambda^2 \int_{b_{\min}}^{b_{\max}} \frac{db^2}{b^2+\lambda^2} = \pi\lambda^2 \ln\left\{\frac{\lambda^2+b_{\max}^2}{\lambda^2+b_{\min}^2}\right\}$$

$$\left\langle \Delta v_{e,\|} \right\rangle = -\frac{e^4}{m_e^2 v_0^4} \frac{v_0}{b_{\max}^2 - b_{\min}^2} \ln\left\{\frac{\lambda^2+b_{\max}^2}{\lambda^2+b_{\min}^2}\right\}$$

And from this using

$$v_{i,e} = n_e v_0 \sigma$$

σ collision cross section

$$\frac{E}{J} = \eta = \frac{e^2}{m_e v_0^3} \frac{\sigma}{b_{\max}^2 - b_{\min}^2} \ln \Lambda$$

- what is the meaning of bmax and bmin and λ ?

a) bmin is the closest approach of the electron, if physical radius of proton

$$b_{\min} \approx 10^{-13} cm$$

b) bmax, maximum for impact parameter,

for $b_{\max} \geq \lambda_D$, (the Debye length) the electron does NOT

feel the presence of the proton.

Hence:

$$b_{\max} = \lambda_D = \left\{ \frac{KT_e}{4\pi n_e e^2} \right\}^{1/2}$$

c) $$|\lambda| = \frac{e^2}{m_e v_0^2}$$

impact parameter for $\sim \pi/2$ deflection.

For corona:

$\lambda_D \approx 0.03 cm$, $\lambda \simeq 3 \times 10^{-10} cm$, $b_{\min} \leq \lambda \leq \lambda_D$

Also collision cross-section $\sigma = \pi b_{\max}^2$ so:

$$\frac{\sigma}{\left(b_{\max}^2 - b_{\min}^2\right)} \simeq \pi$$

$$\Lambda^2 = \frac{\lambda^2 + b_{\max}^2}{\lambda^2 + b_{\min}^2} = \frac{\lambda_D^2}{\lambda^2} = \frac{KT_e}{4\pi n_e e^2} \frac{m_e^2 v_0^4}{e^4}$$

Note that:

$$\frac{1}{2} m_e v_0^2 = \frac{3}{2} KT_e$$

Hence:

$$\Lambda = \left\{ \frac{KT_e}{4\pi n_e e^2} \frac{\left(KT_e\right)^2}{e^4} \right\}^{1/2} = \frac{3\left(KT_e\right)^{3/2}}{\left(4\pi\right)^{1/2} \left(n_e\right)^{1/2} e^3}$$

$$\Lambda = 3 \times 4\pi \lambda_D^3 n_e$$

$$\Lambda = N$$ N: Number of particles

$$= \frac{1}{L}$$ L: The plasma parameter

1. Spitzer Resistivity:

$$\gamma_e(Z=1) = 0.582$$

$$\eta = \frac{\pi^{3/2} m_e^{1/2} Z e^2}{\gamma_R^2 (2K_B T)^{3/2}} \ln\Lambda$$

$$\ln\Lambda = \frac{3}{2Ze^2}\left(\frac{K_B T^3}{\pi n}\right)^{1/2}$$

2. MHD equations in Gaussian units:

Induction:

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{V} \times \vec{B}\right) - c\vec{\nabla} \times \eta \vec{J}$$

$$\frac{\eta c}{4\pi} \nabla^2 \vec{B}$$

Momentum:

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \vec{\nabla} P + \rho \vec{g}$$

Induction simplification: $$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{V} \times \vec{B} \right) + \eta \nabla^2 \vec{B}$$

Magnetic Reynold's number: (Dimensionless)

$$R_m \triangleq \frac{VL}{\eta} \quad \text{(Gaussian)}$$

$R_m \gg 1$: Advection dominates, Close to ideal MHD

$R_m \ll 1$: Diffusion dominates.