Plasma Physics (Lecture-01)

A special kind of fluid in which the constituent particles are charged

$\Rightarrow$ particle in plasma interact through long-range electromagnetic interactions which makes trajectory of individual particle quiet complaicated.

Mass and particle Flux

Continuity equation $\longrightarrow$ Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

$\rho \rightarrow$ Density (mass density) of plasma

$\vec{V} \rightarrow$ Velocity of fluid element

Note:

$\boxed{m\,n\,V}$

m → mass of each particle

n → No. density

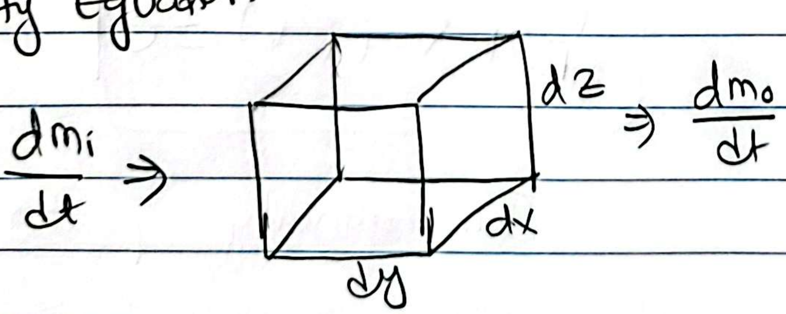
V → velocity of fluid element

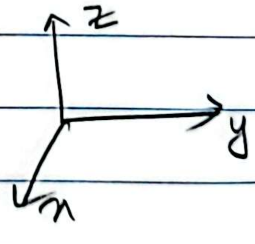
mn → ρ

$\Rightarrow$ mass flux $= mnv = \rho V$

$\boxed{kg/m^2 sec}$

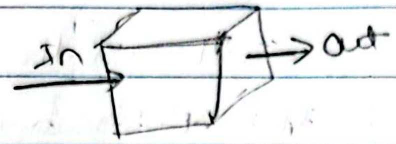
Continuity Equation





Rate of change of mass in the cube = mass flow rate out − mass flow rate in

$$\frac{dm}{dt} = \frac{dm_o}{dt} - \frac{dm_i}{dt}$$



$$\frac{dm}{dt} = \left(\frac{dm_{xo}}{dt} + \frac{dm_{yo}}{dt} + \frac{dm_{zo}}{dt}\right) - \left(\frac{dm_{xi}}{dt} + \frac{dm_{yi}}{dt} + \frac{dm_{zi}}{dt}\right)$$

$$\frac{\partial \rho}{\partial t} dx dy dz = (\rho_{x_o} V_{x_o} - \rho_{x_i} V_{x_i}) dy dz$$
$$+ (\rho_{y_o} V_{y_o} - \rho_{y_i} V_{y_i}) dx dz$$
$$+ (\rho_{zo} V_{zo} - \rho_{zi} V_{zi}) dx dy \quad \text{—— (i)}$$

$$\rho_{x_o} V_{x_o} - \rho_{x_i} V_{x_i} = -\Delta \rho_x V_x$$
$$= -d\rho_x V_x \text{ (for small chang.)}$$

Dividing (i) by $dx dy dz$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho_x V_x)}{\partial x} - \frac{\partial(\rho_y V_y)}{\partial y} - \frac{\partial(\rho_z V_z)}{\partial z}$$

$$\therefore \boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0}$$

The continuity equation

[Fluid velocity dependes on x, y, z as well as t]

$\frac{\partial \rho}{\partial t}$ ⇒ local rate of change of mass density

$\nabla \cdot (\rho V)$ ⇒ Divergence of the mass flux, rate at which mass flows out of a given volume.

General force Equation in fluids

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + F_{external}$$

Here,

$$\frac{dv}{dt} = \underbrace{\frac{\partial v}{\partial t}}_{\text{local}} + \underbrace{(\vec{v} \cdot \nabla)\vec{v}}_{\text{advective change}}$$

$-\nabla p$ ⇒ pressure gradient force

F_{ext} ⇒ External force acting

If gravity is acting (g acts in -ve z-dirn)

$$\rho \frac{dv}{dt} = -\nabla p - \rho g$$

∴ Simplified momentum equation

$$\rho \frac{dv}{dt} = -\rho g - \nabla p$$

① $\rho \frac{dv}{dt}$ ⇒ intertia term ⇒ Describes the change in momentum of the fluid due to acceleration

$-\rho g$ → force due to gravity

$-\nabla p$ → force due to spatial variations in pressure

For static atmosphere $\left(dV/_{dt}=0\right)$

$$0 = -\rho g - {dP}/_{dz}$$

$$\Rightarrow \frac{dP}{dz} = -\rho g \text{ (Hydrostatic equilibrium)}$$

$P \rightarrow$ pressure (varies with z)
$\rho \rightarrow$ mass density of air (varies with z)
$g \rightarrow$ acceleration due to gravity
$z \rightarrow$ Altitude

Relation of density and pressure

$$P = 2\rho RT$$

$$\rho = \frac{P}{2RT}$$

$$\frac{dP}{dz} = -\frac{P}{2RT}g$$

$$\frac{dP}{P} = -\frac{g}{2RT}dz + c$$

$$\log P = -\frac{g}{2RT}z + c$$

$$\Rightarrow \boxed{P = P_0 e^{-z/H}}$$

P_0 : pressure at sea level ($z=0$)

$H = \frac{2RT}{g}$: Scale factor

Height over which the pressure decreases by a factor of e

$R \approx 287.J/kg.k$

$T \approx 288K$ (Global.avg.),

$g = 9.81 m/s^2$

$R = 8.314\ J/mol.k$

For gas on earth Average molar mass

$= 28.96$ grams per mole

$$H = \frac{2 \times 287 \times 288}{9.81} = 16.85 km$$

$$R = \frac{8.314}{0.02896} = 287\ J/kg.K$$

⇒ The atmospheric pressure decreases by a factor of e every 16.85Km of altitude

Mean Free Path

The average distance travelled by a particle between two collisions is known as mean free path