

Day 02 (lecture notes)

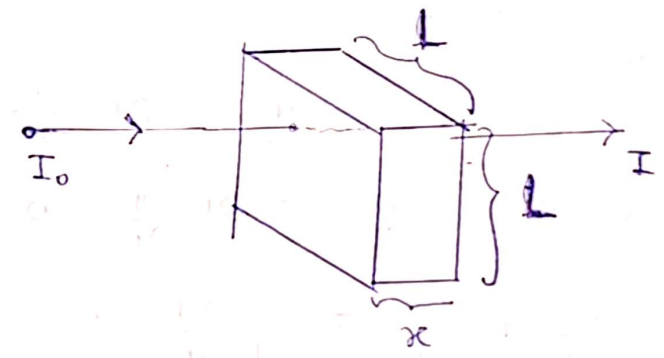
Wiki-Source : scattering theory

$$T = \frac{I}{I_0} = e^{-x/l}$$

T: fraction of particles that pass through

$$I = I_0 e^{-x/l} \quad (\text{Beer-Lambert law})$$

T: transmission



* Mean free path is average distance over which a moving particle travels without collision $l = (\sigma n)^{-1}$

n: particle density of the target

Wik Source : Kinetic theory of gas

$$\text{relative velocity: } v_{re} = \sqrt{2}v$$

σ : effective cross-section area

↓
lead to the table about vacuum range

* if mean free path \leq molecular size (ex: 0.3nm for N₂)

\Rightarrow nuclear physics

* In plasma particles interact before collision due to charges.

Solar corona

Pressure distribution of solar corona due to hydrostatic equilibrium is given by

$$\frac{P}{P_0} = \exp\left\{-\frac{r_0}{H}\left(1 - \frac{r_0}{r}\right)\right\}$$

r : radius

r_0 : radius of base of the corona

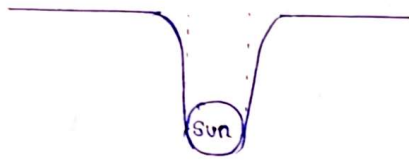
H : height

$$H = \frac{2k_B T_0 r_0^2}{G M m_p}$$

M : mass of the Sun

m_p : mass of a proton

T_0 : temp at base of the corona



gravitational well

* If particles have enough thermal energy they can escape the gravitational potential.

Parker Solar wind equation

Continuity equation $\frac{\partial}{\partial r}(\rho v r^2) = 0 \implies \rho v r^2 = 0 \implies \textcircled{1}$

Momentum equation $\rho v \frac{\partial v}{\partial r} = -\frac{G M_0 \rho}{r^2} - \frac{\partial P}{\partial r} \implies \textcircled{2}$

gas law $P = \rho C_s^2 \implies \textcircled{3}$

C_s^2 : Critical velocity
Sun's velocity
radial velocity

$$\frac{\partial}{\partial r}(\rho v r^2) = 0$$

$$\frac{\partial \rho}{\partial r} \cdot v r^2 + v \rho \cdot \frac{\partial r^2}{\partial r} + \rho r^2 \frac{\partial v}{\partial r} = 0$$

$$v r^2 \frac{\partial \rho}{\partial r} + 2 v \rho r + \rho r^2 \frac{\partial v}{\partial r} = 0$$

$$\frac{1}{\rho} \frac{\partial \rho}{\partial r} + \frac{2}{r} + \frac{1}{v} \frac{\partial v}{\partial r} = 0 \implies \textcircled{4}$$

$\textcircled{2}, \rho v \frac{\partial v}{\partial r} = -\frac{G M_0 \rho}{r^2} - \frac{\partial P}{\partial r}$; $\textcircled{3}, P = \rho C_s^2$

$$v \frac{\partial v}{\partial r} = -\frac{G M_0}{r^2} - \frac{1}{\rho} \cdot C_s^2 \cdot \frac{\partial \rho}{\partial r} \implies \textcircled{5}$$

look the note in the web page for the derivation

* Bernoulli type equation for steady flow

$\ln(\)$ term are logarithmic variations

← Kinetic Energy

← gravitational potential

$$\frac{v^2}{2 C_s^2} - \frac{1}{2} \ln(v^2) = \frac{G M_0}{C_s^2 r} + 2 \ln(r) + \text{constant} \quad ; \text{ Parker's solar wind equation}$$

; v^2 can be + or - $\implies \therefore$ mass flow can be in or out
in: black hole

out: solar wind.

solution

graph (note) \rightarrow only v and is related to solar wind

Critical point (Sonic point): here radial velocity equal to critical sound speed. here flow turn in to supersonic from subsonic and accelerate away from the sun.

Next week

Governing equations:

Continuity equation : $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$ (mass conservation)

Momentum equation : $\frac{\partial (\rho v)}{\partial t} + \nabla (\rho \cdot v^2) = -\nabla P + \rho g + F_{\text{other}}$
(Navier stock law)

Energy equation (first law of thermody.-) : $\frac{\partial E}{\partial t} + \nabla \cdot ((E + P)v) = \frac{\dot{Q} - L}{\Delta Q}$

Maxwell equations : $\nabla \cdot B = 0$; $\nabla \cdot E = \rho / \epsilon_0$
 $\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times (\eta \nabla \times B)$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Faraday's law : $\nabla \times E = -\frac{\partial B}{\partial t}$