Note on Debye length and plasma parameter

Debye Length

Introduction

The term plasma was introduced by Tonks and Langmuir (1929). The term 'plasma' is to describe a region of an arc discharge where ion and electron densities are high and nearly equal. They compare the behaviour of discharge to blood, with ions acting like a rigid jelly when electrons oscillate. plasma now refer to assemblies of charged Particle studied in plasma physics, where matter both behaves as particle and fluid. This is similar to ~~Where matter behaves both as Particles~~ how quantum mechanics reveals dual particle-wave properties.

The discussion adopts the gaussian unit system for electromagnetism, where electric and magnetic quantities are measured in electrostatic and electromagnetic units respectively.

From Maxwell's eq^n

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \quad - (1)$$

$$\nabla \cdot B = 0 \quad - \quad (2)$$

$$\nabla \cdot E = 4\pi \rho \quad - \quad (3)$$

$$\nabla \times B = \frac{1}{c}\frac{\partial E}{\partial t} + 4\pi j \quad - \quad (4)$$

Where ρ is charge density, J is current density, E is electric field and B is magnetic field.

So, from eqⁿ (1) and (2)

$$\frac{1}{c}\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 \quad - \quad (5)$$

let us consider simple plasma composed of fully ionized hydrogen. Then mass density, charge density and current density is given by

Mass density $(\rho) = n_e m_e + n_i m_i$

charge density $(\xi) = e(n_i - n_e)$

Current density $(j) = \frac{e}{c}(n_i v_i - n_e v_e)$

Where, $n_{e,p}$ = no. ~~of~~ density of electron and Proton

$m_e m_p$ = mass of electron and Proton

$-e, p$ = charge of electron and Proton.

let us consider a plasma with initially uniform proton and electron densities, with no net charge density, so initially electric field is zero. let n_0 be initial densities ~~of~~ of Proton and electron. let us Suppose proton density change from n_0 to $(1-\delta)n_0$ in region $-L < x < L$. If L is sufficiently small the electric field due to this change will be small so, electron will suffer negligible distribution. If L is very Large then distribution of electron will be very huge. If we take value of L ~~so~~ in such a way that just transition takes place then equ^n (3) becomes

$$\frac{d^2\phi}{dx^2} = 4\pi\delta n_0 e \quad - (6)$$

Consider that plasma as a whole is maintained ~~at~~ potential $\phi=0$, then equ^n (6) becomes

$$\left.\begin{array}{ll} \phi = 2\pi\delta n_0 e(x^2-L^2), & |x|<L \\ \phi = 0 & |x|>L \end{array}\right\} - (7)$$

so,

$$\phi(0) = -2\pi\delta n_0 e L^2 - (8)$$

consider, plasma has Temp^r T, K.E. is $\frac{1}{2}KT$ in each degree of freedom. If $\phi(0)$ is so small that avg. thermal energy $\simeq 0$, then there will be only small change in state of plasma. If $\phi(0)$ is very large then only ~~small~~ few electron can reach $x=0$ So, for 'quasi neutral' state

$$\frac{1}{2}KT > 2\pi\delta n_0 e^2 L^2 \quad - (9)$$

Also,

$$\delta < (\lambda_D/L)^2, \quad \lambda_D = \frac{KT}{4\pi n e^2} \quad - (10)$$

Also, For Plasma Parameter (Λ)

$$\therefore \lambda_D = \sqrt{\frac{KT}{4\pi n e^2}} \quad \text{--- (11)} \quad \text{OR} \quad \lambda_D = \sqrt{\frac{\epsilon KT}{ne^2}}$$

W

$$\boxed{\therefore \lambda_D = 10^{0.84} n^{-1/2} T^{1/2}}$$

Where,

λ_D is debye length

If,

(1) $\delta = -\lambda_D$ proton is completely removed from $-L < x < L$, plasma in quasi neutral ($L \ll \lambda_D$)

(2) If $L \gg \lambda_D \rightarrow$ behaves as vaccum.

Height Scale

let us consider solar corona, in which $m_p \ggg m_e$. gravitational field only acts on proton only. Due to gravitational field densities decreases exponentially with height, Which is given by

$$\boxed{H = \frac{KT}{m_{av} \cdot g}}$$

m_{av} avg. Particle mass $\simeq \frac{1}{2} m_p$

g is gravi- acceleration.

Ref:- Plasma phy by: Peter A. STURROCK
- Arnab R. choudhori

Also, For Plasma Parameter (Λ)

$$\boxed{\Lambda = \frac{4\pi}{3} \lambda_D^3 n_e}$$

Where, $\Lambda_e = n_e \lambda_{De}^3$, $\Lambda_s = n_s \lambda_{Ds}^3$

i.e $\Lambda_e = \left(\frac{K}{4\pi e^2}\right)^{3/2} n_e^{-1/2} T_e^{3/2}$

(1)

HW - 3

Calculate the numerical value of the Debye length and Plasma Parameter in

(a) the solar corona ($T - 10^6 K$, $n_e \sim 10^{14} m^{-3}$)

(b) the earth magnetosphere ($T - 10^7 K$, $n_e \sim 10^7 m^{-3}$)

(c) the interior of the sun ($T \sim 10^{32} K$, $n_e \sim 10^7 m^{-3}$)

(d) the interior of White draft ($10^7 K$, n_e - 29-32 m^{-3})

(e) the interior of a neutron star. (~~36^K to $38K$~~, n_e ~~4K~~ $10^{11} - 10^{13} K$ { $10^{36} - 10^{40} m^{-3}$)

Solⁿ

$$\lambda_D (\text{solar corona}) = \sqrt{\frac{\varepsilon K T}{n e^2}}$$

Where

$$\varepsilon = 8.85 \times 10^{-12} F m^{-1}$$

$$K_B = 1.38 \times 10^{-23} J K^{-1}$$

$$\lambda_D = \sqrt{\frac{8.85 \times 10^{-12} \times 10^6 \times 1.38 \times 10^{-23}}{10^{15} \times (1.67 \times 10^{-19})^2}}$$

$$= \sqrt{4.7 \times \cancel{10^{-5}}\ 10^{-6}\ \cancel{m}} = 2.18 \times 10^{-3} m = 0.218 cm$$

$$\lambda_D = 2.18 m = 0.218 cm.$$

$$\therefore \lambda_D = 0.218 cm$$

Plasma Parameter $(\Lambda) = \frac{4\pi}{3} \lambda_D^3 \cdot n_e$

$$= \frac{4\pi}{3} \times (2.18 \times 10^{-3})^3 \times 10^{15}$$

$$= 4.3 \times 10^7$$

(b) Earth magnetosphere $(T \sim 10^7 K,\ n_e \sim 10^7 m^{-3})$

$$\lambda_D = \sqrt{\frac{\varepsilon K_B T}{n_e e^2}}$$

$$= \sqrt{\frac{8.85 \times 10^{-12} \times 1.38 \times 10^{-23} \times 10^7}{10^7 \times (1.69 \times 10^{-19})^2}}$$

$$= \sqrt{\frac{120.213 \times 10^3}{2.8561}}$$

$$= \sqrt{4.2 \times 10^3 m}$$

$$= 64.80 m$$

Plasma Parameter $(\Lambda) = \frac{4\pi}{3} \lambda_D^3 \cdot n_e$

$$= \frac{4\pi}{3} \cdot (64.80)^3 \cdot 10^7$$

$$= 1.13 \times 10^{13}$$

(c) the interior of the sun $(T \sim 10^{7} K, n_e \sim 10^{30} m^{-3})$

$$\lambda_D = \sqrt{\frac{8.85 \times 10^{-12} \times 10^{7} \times 1.38 \times 10^{-23}}{10^{30} \times (1.69 \times 10^{-19})^2}}$$

$$= 6.91 \times 10^{-10} m.$$

$$\text{Plasma Parameter}(\Lambda) = \frac{4\pi}{3} \lambda_D^3 \cdot n_e$$

$$= \frac{4\pi}{3} \times (6.91 \times 10^{-10})^3 \cdot 10^{7}$$

$$= 1.38 \times 10^{-20}$$

(d) The interior of White draft

$$\lambda_D = \sqrt{\frac{8.85 \times 10^{-12} \times 10^{7} \times 1.38 \times 10^{-23}}{10^{32} \times (1.69 \times 10^{-19})^2}}$$

$T = 10^{7} K$
$n_e \sim 10^{32}$

$$= \sqrt{\frac{8.85 \times 1.38 \times 10^{-12+7-23}}{(1.69)^2 \times 10^{-38+32}}}$$

$$= \sqrt{\frac{1.22 \times 10^{-27}}{2.56 \times 10^{-6}}} = 6.69 \times 10^{-11} m$$

$$\text{Plasma Parameter}(\Lambda) = \frac{4\pi}{3} (\lambda_D)^3 n_e$$

$$= 1.38 \times 10^{2}$$

H.W # 4

We have

Momemtum equ^n for electron

$$m_e n \frac{d\vec{V_e}}{dt} = nq\left(\frac{\vec{V_e}\times\vec{B}}{c}\right) + nqE - \vec{\nabla} q_e \quad (1)$$

momemtum equ^n for Proton

$$m_p n \frac{d\vec{V_p}}{dt} = -nq\left(\frac{\vec{V_p}\times\vec{B}}{c}\right) - nq\vec{E} - \nabla q_p \quad (2)$$

We know that, som of

Total pressure of gas is equal to pressure enerted by electron and Proton

i.e $P = P_e + P_p$ — (3)

from the defination of current density

$(J) = qn_e\vec{v_e} - qn_p\vec{V_p}$ — (4)

now, Add equ^n (1) and (2)

$$n\left(m_e\frac{d\vec{V_e}}{dt} + m_p\frac{d\vec{V_p}}{dt}\right) = nq\left(\frac{\vec{V_e}\times\vec{B}}{c} - \frac{\vec{V_p}\times\vec{B}}{c}\right) \quad (5)$$

Since, Pressure enerted by electron is very small as compare to pressure enerted by Proton so from equ^n (5) we neglect mass of electron term.

$$m_p \frac{dv_p}{dt} = \vec{J} \times \frac{\vec{B}}{c} - \vec{\nabla} p - ⑥$$

Also, We have relation

$$n\vec{v} = \frac{n_e m_e \times v_e + n_p m_p \times v_p}{m_e + m_p}$$

dividing by m_e eq

$$nv = n_p v_p \quad - ⑦$$

So, eq^n ⑥ can be written as

$$\boxed{\rho \frac{d\vec{v}}{\partial t} = \vec{J} \times \frac{\vec{B}}{c} - \nabla p} \quad - ⑧$$

from substracting eq^n ① - ⑪, we get

$$\boxed{\cancel{\nabla} \times \frac{\vec{B}}{c} + \vec{E} = 0} \quad - ⑨$$

by using eq^n ⑨

$$E = -(\nabla \times B) \quad - ⑩$$

Substituting ⑩ in ②

$$m_n \frac{d\vec{v}}{dt} = -nq\left(\frac{v \times B}{c}\right) + nqE - \nabla p$$

$$m_n \frac{d\vec{v}}{dt} = \cancel{-nq\left(\frac{v \times B}{c}\right)} + \cancel{nq(v \times B)} - \nabla p$$

$$- ⑪$$

$$\boxed{m_n \frac{d\vec{v}}{dt} = -\nabla p} \quad - \text{momemtum eq}^n.$$

Mass conservation equ^n

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0} \quad -(12)$$

(3) Induction equ^n

$$\frac{\partial B}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) \quad (\text{from } 10)$$

$$\boxed{\frac{\partial B}{\partial t} = -\vec{\nabla}\times\vec{E}} \quad -(13)$$

So, MHD equ^n with elimating electric field are of equ^n (11) (12) and (13)