Feb 19

$$\beta \ll 1 \Rightarrow \vec{j} \times \vec{B} = \vec{f}_L = 0$$

$$\vec{j} = \alpha \vec{B}$$

$$\vec{\nabla}\alpha \cdot \vec{B} = 0$$

α is function of space

$$\alpha(\vec{r}) = \alpha_0 = \text{const}$$

$$\vec{\nabla} \times \vec{B} = \alpha_0 \vec{B}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \alpha_0 (\vec{\nabla} \times \vec{B})$$

$$\Delta \vec{B} = \alpha_0^2 \vec{B}$$

↳ cot $\boxed{\vec{\nabla} \cdot \vec{B} = 0}$

Note: If $\operatorname{div}(\text{sth}) = 0$

curl of curl( ) =

Cartesian Solution (General)

$$\vec{B} = \vec{B}_0 \, e^{-i(k_x x + k_y y + k_z z)}$$

$$k^2 + \alpha_0^2 = 0$$

$\boxed{\text{Helmholtz equation}}$

NOTE

> Happens in some tube after the disruption
> α_0 = const solution

Concept of Helicity

Introducing Vector potential

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$

Helicity definition ⇒

$$\text{Helicity} = H = \int (\vec{A}\cdot\vec{B})\, dV$$

⇓

Concept introduced by Gauss

Ideal MHD

- No dissipation
- low Reynold's no
- No. of windings is preserved

↓

$$\vec{A}' = \vec{A} + \vec{\nabla}\varphi$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{A}' = \vec{\nabla}\times\vec{A}}$$

$$\vec{B}' = \vec{B}$$

Now,

$$\vec{H}' = \int \vec{A}' \cdot \vec{B} \, dV$$

$$= \int \vec{A} \cdot \vec{B} \, dV + \int \nabla\varphi \cdot \vec{B} \, dV$$

$$\boxed{\vec{H}' - \vec{H} = \int \vec{\nabla}\varphi \cdot \vec{B} \, dV}$$

$$\vec{\nabla} \cdot \varphi \vec{B} = \vec{\nabla}\varphi \cdot \vec{B} + \varphi \underbrace{\vec{\nabla} \cdot \vec{B}}_{0}$$

$$\Rightarrow \vec{\nabla} \cdot \varphi \vec{B} = \vec{\nabla}\varphi \cdot \vec{B}$$

Now,

$$H' - H = \int \vec{\nabla} \cdot \varphi \vec{B} \, dV$$

$$= \int (\varphi \vec{B}) \cdot d\vec{\sigma}$$

$= 0$

While taking volume integral if field lines doesn't go at any angle then Helicity is conserved.

$\boxed{\text{In Ideal MHD}}$

Ideal MHD Equation

$$\frac{\partial B}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

$\Downarrow$

No dissipiation

For Helicity

$$H = \int \vec{A} \cdot \vec{B} \, dV$$

$$H = \int_V \vec{A}\cdot\vec{B}\,dV$$

Taking time derivative

$$\frac{dH}{dt} = \frac{\partial}{\partial t}\int_V \vec{A}\cdot\vec{B}\,dV$$

$$= \int_V \left(\frac{\partial \vec{A}}{\partial t}\cdot\vec{B} + \vec{A}\cdot\frac{\partial \vec{B}}{\partial t}\right)dV$$

$$\vec{B}\cdot\vec{\nabla}\times\vec{A}$$

$$= \int_V \left[\frac{\partial \vec{A}}{\partial t}\cdot\vec{B} + \underbrace{\vec{A}}_{}\cdot\underbrace{\vec{\nabla}}_{}\times\underbrace{\left(\vec{\nabla}\times\vec{B}\right)}_{}\right]dV$$

Formula book (5) (Pg. 4)

$$\underset{a}{\vec{B}} \cdot \vec{\nabla} \times \underset{v \times n}{\vec{A}} = \vec{\nabla} \cdot (\vec{A} \times \vec{B}) + \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$= \int \frac{\partial \vec{A}}{\partial t} \cdot \vec{B} \, dV + \int \vec{\nabla} \cdot (\vec{v} \times \vec{B} \times \vec{A}) \, dV$$

$$+ \vec{v} \times \vec{B} \cdot \cancel{(\vec{\nabla} \times \vec{A})} \, dV$$

$$= \int \frac{\partial \vec{A}}{\partial t} \cdot \vec{B} \, dV + \int_S \vec{v} \times \vec{B} \times \vec{A} \, d\vec{S}$$

0

very far from the surface

$$+ \int \cancel{\left(\vec{V} \times \vec{B}\right)} \cdot \vec{B} \, dV$$

$$0$$

$$\frac{dH}{dt} = \int_V \frac{\partial \vec{A}}{\partial t} \cdot \vec{B} \, dV$$

$$= \int_V \vec{V} \times \vec{B} \cdot \vec{B} \, dV$$

$$= 0$$

which shows that

helicity is conserved.

Helicity is conse

(far away from the sun)