Scale Height in Atmospheres

Hello Folks,

I noticed several of you have had problems with the concept of scale Height, so Here is some Background

1) Scale Height in Flat, parallel Atmosphe

a) $\frac{\partial P}{\partial z} = -\rho g$; g = gravitational Acceleration at surface

b) $P = \rho C_s^2 = \frac{\rho k T}{\mu m_H}$; Dep. of sound speed

Assume T = constant $\Rightarrow C_s^2$ = constant

$H \triangleq C_s^2/g$

$\Rightarrow \frac{\partial P}{\partial z} = -\frac{P}{H} \Rightarrow P(z) = P_0 e^{-z/H}$

This also applies for $z \ll r_0$

2) Now go to a spherical geometry

$g \hat{=} \frac{GM}{r_0^2}$ (r_0 = surface)

$H = C_s^2/g = \frac{C_s^2 r_0^2}{GM}$

Atmospheric Scale Height

2) continue

$\frac{\partial P}{\partial r} = -\rho \frac{GM}{r^2} = -\frac{P}{C_s^2}\frac{GM}{r^2} = \frac{P}{H}\frac{r_0^2}{r^2}$

Solve: $\frac{P}{P_0} = \exp -\left[\frac{r_0}{H}\left\{1 - \frac{r_0}{r}\right\}\right]$

So, clearly, there is only one relevant parameter here $r_0/H \hat{=} \lambda$

In most cases $r_0 \gg H$

e.g. Sun: $H \simeq 10^{10}$ cm, $r_0 = 7\times10^{10}$ cm

Mars: $H \simeq 1.5\times10^6$ cm $r_0 = 3.4\times10^8$ cm

So Sun $\rightarrow \lambda \simeq 7$, Mars, $\lambda \simeq 200$

That's all there is to it!